A Model of
Monetary Policy and Risk Premia

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Abstract

We develop a dynamic asset pricing model in which monetary policy affects the risk premium component of the cost of capital. Risk-tolerant agents (banks) borrow from risk-averse agents (i.e. take deposits) to fund levered investments. Leverage exposes banks to funding risk, which they insure by holding liquidity buffers. By changing the nominal rate the central bank influences the liquidity premium in financial markets, and hence the cost of taking leverage. Lower nominal rates make liquidity cheaper and raise leverage, resulting in lower risk premia and higher asset prices, volatility, investment, and growth. We analyze forward guidance, a "Greenspan put", and the yield curve.

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I. Introduction

In textbook models (e.g. Woodford, 2003), monetary policy works by changing the real interest rate. Yet a growing body of empirical evidence shows that monetary policy also has a large impact on the *risk premium* component of the cost of capital.[1] Moreover, many central bank interventions can be usefully interpreted as targeting risk premia. For instance, a "Greenspan put" in the 1990s and low interest rates in the mid 2000s arguably led to excessive leverage and compressed spreads.[2] During the financial crisis, large-scale asset purchases, equity injections, and asset guarantees were all explicitly aimed at supporting risky asset prices (see Bernanke, 2013, for a discussion). In subsequent years, with spreads near historic lows, an important debate centers on whether low interest rates fuel "reaching for yield" and hence pose a threat to financial stability (Stein, 2014). These observations point to an underlying risk premium channel of monetary policy.

In this paper, we develop a dynamic asset pricing model of the risk premium channel of monetary policy. In the model, taking leverage exposes financial institutions to funding shocks that require them to liquidate assets. To avoid engaging in costly fire sales, they hold buffers of liquid securities, which can be sold rapidly at full value. Consequently, the cost of taking leverage depends on the cost of holding liquid securities, the liquidity premium. The central bank governs this liquidity premium by varying the nominal interest rate. A low nominal rate leads to a low liquidity premium, a relationship with strong empirical support. A low liquidity premium decreases the cost of taking leverage and hence increases risk taking, which reduces risk premia and the cost of capital in the economy.

Our model features an economy populated by two types of agents who differ in their risk aversion. We think of the more risk tolerant agents as pooling their wealth into the net worth (equity capital) of financial institutions, or banks for short. In equilibrium, banks take levered positions in risky assets by borrowing from the more risk averse agents using short-term risk-free claims, which we think of as taking deposits. Our view of banks as levered risk-takers is purposely simplified, abstracting from other functions such as screening and monitoring in order to focus on risk taking and risk premia. It has the advantage of accommodating a diverse set of financial institutions, including commercial banks, broker-dealers, and hedge funds, whose unifying characteristic is that they take leverage using short-term debt.

Taking deposits exposes banks to funding (rollover) risk (e.g. Allen and Gale, 1994). When hit by a funding shock, banks are forced to redeem a fraction of their deposits. To do

[1] Bernanke and Kuttner (2005) show that monetary policy surprises have a large impact on stock prices and that this impact primarily reflects changes in risk premia. Hanson and Stein (2014), and Gertler and Karadi (2014) find parallel results for long-term bond yields and credit spreads. Gilchrist and Zakrajšek (2012) find that changes in risk premia have a strong influence on the macroeconomy.

[2] See for example Blinder and Reis (2005), Rajan (2011), and Yellen (2011).

so, they must immediately liquidate some of their assets. Liquidating risky assets rapidly is costly because it leads to fire sales. To avoid this, banks hold buffer stocks of liquid securities, which can be liquidated rapidly at full value. Thus, to insure against losses in the event of a funding shock, banks set aside a fraction of each deposit dollar they raise and hold it in liquid securities. In this way, the risk of funding shocks creates a complementarity between holding liquidity and taking leverage.

We model two types of liquid securities: central bank reserves, which have the highest level of liquidity, and government bonds. Banks' demand for liquidity buffers causes liquid securities to command a premium in equilibrium. This liquidity premium depends on the nominal interest rate. The liquidity premium of reserves equals the nominal rate because that is the opportunity cost of holding them. The liquidity premium of government bonds is likewise proportional to the nominal rate because government bonds and reserves are substitutable sources of liquidity. Therefore, by changing the nominal rate, the central bank changes the cost of holding all liquid securities.[3]

Figure 1 examines this prediction empirically. It plots the fed funds rate, a measure of the nominal interest rate, against the spread between the fed funds rate and the three-month T-bill rate, a measure of the liquidity premium on government bonds, from 1955 to 2010. As the figure shows, the relationship between these two series, a rate and a spread, is very strong. Their correlation is 78%, and they exhibit tight comovement both in the cycle and in the trend, consistent with the transmission of the nominal rate to the liquidity premium that we model.

The central bank's ability to influence risk taking works through this transmission mechanism. When the central bank raises the nominal rate, the higher liquidity premium increases banks' cost of taking leverage and so reduces their risk taking. The result is a decrease in the overall demand for risk taking in the economy, an increase in the effective aggregate risk aversion, and ultimately a rise in risk premia.

Monetary policy in our model takes the form of a nominal interest rate rule which is a function of the single state variable, the share of banks' net worth of the total wealth in the economy. We consider a number of interest rate rules and analyze their positive implications for equilibrium prices and quantities.[4]

[3]Recently, interest on reserves has attracted significant attention. When reserves pay interest, reserves' liquidity premium equals the difference between the nominal rate and their rate of interest. In this case, the central bank targets this difference rather than the full level of the nominal rate. Section IV.B provides further discussion.

[4]We do not take a stance on optimality because doing so requires making two strong assumptions. The first is the choice of welfare criterion, which is difficult in our incomplete-markets heterogeneous-agent setup. The second, and more important assumption, requires choosing among the numerous frictions that the literature has argued create a need for regulating leverage, including moral hazard arising from deposit insurance (e.g. Keeley, 1990), pecuniary externalities due to financial constraints (e.g. Stein, 2012), and spillover effects to the oeconomy (e.g. Farhi and Werning, 2013). To preserve the model's wide applicability and transparency,

We begin by analyzing results from the baseline version of the model, which is set in an endowment economy where the risky asset is a claim on the aggregate consumption stream. We compare outcomes under two monetary policy rules, one where the nominal rate is high and one where it is low. To do so, we solve the model numerically using global projection methods that capture its inherent nonlinearities.

Our results show that under the high-rate policy bank leverage is low, and the price of risk (Sharpe ratio) and risk premium are high. In contrast, the real rate is low because the economy's high effective risk aversion increases the precautionary demand for savings. Overall, the effect on the risk premium dominates, so that under the high-rate policy discount rates are substantially higher and valuations (price-dividend ratios) are substantially lower than under the low-rate policy. The difference is largest at moderate levels of the bank net worth share, because risk sharing there is highest as banks are both large enough to influence asset prices, yet small enough to take high leverage.

We also show that monetary policy affects volatility. In particular, low nominal rates lead to higher volatility in the long run. The reason is that low nominal rates lead banks to take greater leverage, which makes their net worth more volatile and hence increases the volatility of discount rates. We further show that under a low-rate policy the stationary distribution of banks' net worth is characterized by both a higher mean and a much higher dispersion than under a high-rate policy. The higher dispersion implies that low nominal rates result in occasional periods of low bank net worth and depressed asset prices, as in financial crises.

To implement its desired nominal rate rule and the liquidity premium it implies, the central bank must ensure that the aggregate liquidity supply evolves as required. We show that the central bank can produce the necessary shifts in the liquidity supply via open market operations, exchanging reserves for government bonds so that the liquidity supply contracts (expands) as required in response to a nominal rate increase (decrease). The required changes in the liquidity supply can also be carried out by shifts in the supply of liquid assets produced by the private sector. Indeed, as Drechsler, Savov, and Schnabl (2014) show, increases in the nominal rate induce big inward shifts in the supply of retail bank deposits, a large and important class of liquid assets. For simplicity, in the model we subsume such adjustments into the evolution of the supply of "government bonds". Either way, by changing the nominal rate the central bank affects the liquidity premium, as shown in Figure 1, and the model's implications follow from this relationship.

We further examine the impact of the nominal rate on banks' holdings of liquid assets. As the nominal rate decreases, so does the cost of liquidity, and banks' holdings of liquid assets grow. Indeed, when the nominal rate is near zero, banks hold large amounts of liquid assets.

we do not settle on any one of these arguments and therefore do not conduct welfare analysis.

The central bank achieves this low cost (equivalently, relatively high return) to holding liquid securities by ensuring that their nominal supply grows slowly, a positive illustration of the Friedman rule (1969).[5]

Nominal rates in the model are naturally bounded below by zero. They cannot fall below zero as this would create an arbitrage in which banks raise deposits to invest in liquid securities. Yet even when rates are at the zero lower bound the central bank can further support asset prices by using forward guidance. Under forward guidance, the central bank commits to keeping nominal rates low further into the future, even after the economy recovers (banks' net worth rises above some threshold). We show that forward guidance causes asset prices to rise in anticipation of lower future discount rates. The effect on asset prices peaks as the nominal rate nears liftoff. In this region a small change in the timing of anticipated rate increases can provoke a large response in asset prices (a market "tantrum").

In a second application of a dynamic policy, we analyze the effects of a "Greenspan put" policy in which the central bank progressively cuts rates following a large-enough sequence of negative shocks. We show that a Greenspan put is effective at stabilizing asset prices locally by boosting bank leverage. However, as ever-increasing leverage cannot be sustained indefinitely, further negative shocks cause prices to fall drastically and volatility to surge. Thus, in our framework a Greenspan put reduces volatility in the short run at the expense of potentially greater instability in the long run.

We further analyze how the model's asset pricing implications affect macroeconomic outcomes. To do so, we extend the baseline model in two ways. The first is by adding production, which allows us to look at investment and economic growth. The second is by introducing a persistent shock to the nominal interest rate that is independent of other shocks in the economy. Such nominal rate shocks make our analysis comparable to the monetary economics literature. They also enrich our results by producing balance sheet amplification effects (Bernanke, Gertler, and Gilchrist, 1999).[6] As is common in the literature, we present the results of the extended model in the form of impulse response functions following a shock to the nominal rate from the model's stochastic steady state. These are computed without linearizing, using a global solution for the extended model.

Consistent with the baseline model, a positive nominal rate shock causes bank leverage and net worth to fall, and risk premia and Sharpe ratios to rise. It also causes the price of capital to fall, which leads to a drop in investment. The result is that economic growth slows and output ends up at a permanently lower level. These results demonstrate that the risk

[5]It is worth highlighting that it is in fact the growth rate rather than the number of liquid securities that determines their holding period return and hence the nominal interest rate. For instance, a one-off doubling of liquid securities simply doubles the price level, whereas a one percentage point increase in their expected growth rate raises the nominal interest rate by one percent.

[6]That is, a surprise rate hike raises risk premia, which reduces asset prices, which reduces banks' net worth, which raises risk premia more, which reduces asset prices more, and so on.

premium channel has strong implications for aggregate economic activity, which is consistent with the results of Gilchrist and Zakrajšek (2012).

We conclude the analysis by computing the term structure of nominal interest rates implied by our extended model. The model generates an upward-sloping term structure and a large positive term premium, as high nominal interest rates are associated with a high liquidity premium and hence lower risk sharing and higher marginal utility. Moreover, the term premium rises following a positive shock to the nominal rate, consistent with the results of Hanson and Stein (2014).

The rest of the paper is organized as follows: Section II reviews the literature, Section III lays out the baseline model, Section IV characterizes the equilibrium, Section V presents the results, Section VI lays out the extended model and presents corresponding results, and Section VII concludes.

II. Related literature

Our paper is related to the literature on the bank lending channel of monetary policy developed by Bernanke (1983), Bernanke and Blinder (1988), Kashyap and Stein (1994), Bernanke and Gertler (1995), and Stein (1998, 2012). This literature relies on a reserve requirement: a contraction in reserves forces banks to shrink their deposits and hence their assets since deposits cannot be easily replaced. In contrast, in our framework an increase in the nominal rate reduces financial institutions' willingness to take risk. This is because it results in an increase in the liquidity premium, which makes it more costly to insure against a loss of funding. Our model thus applies beyond traditional banks to all types of levered institutions, such as broker-dealers, hedge funds, and off-balance sheet vehicles, and our focus extends beyond bank loans to risk premia and asset prices more broadly.

Our paper also relates to the literature on the balance sheet channel (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997) in which monetary policy shocks affect the net worth of borrowers and hence their ability to raise capital and invest. Bernanke, Gertler, and Gilchrist (1999), Jermann and Quadrini (2012), and Christiano, Motto, and Rostagno (2014) embed the balance sheet channel into quantitative macro models. Since the financial crisis, attention has shifted from firms' balance sheets to those of financial intermediaries (He and Krishnamurthy, 2011, 2013; Brunnermeier and Sannikov, 2014a). A maturity mismatch between assets and liabilities causes intermediary net worth to fall when interest rates rise unexpectedly, which forces balance sheets to contract (e.g. Cúrdia and Woodford, 2009; Adrian and Shin, 2010; Gertler and Kiyotaki, 2010; Adrian and Boyarchenko, 2012). In Brunnermeier and Sannikov (2014b), such balance sheet shocks cause banks to contract money creation, which leads to deflation and further contraction. In our model unexpected rate changes also act as an amplifier. However, our model differs from the literature in that it establishes a relationship between the level of the nominal rate and the level of risk premia, which is important for analyzing the effects of a low-rate environment on asset prices and financial stability.

Our modeling of funding risk and its effects on risk taking has its roots in the banking literature, which emphasizes the liquidity transformation role of the banking sector (Diamond and Dybvig, 1983; Gorton and Pennacchi, 1990; Shleifer and Vishny, 1997; Kashyap, Rajan, and Stein, 2002). Banks hold assets that are prone to fire sales and issue short-term safe liabilities against them. The resulting liquidity mismatch exposes banks to funding risk, leading them to demand liquid assets as insurance (Bhattacharya and Gale, 1987; Allen and Gale, 1994). As shown by Holmström and Tirole (1998), the supply of liquidity by the private sector is in general insufficient, which creates an important role for public liquidity provision. Our model highlights how, by varying the supply of liquid assets, monetary policy can have a powerful impact on risk taking, and hence risk premia and asset prices.

On the methodological side, our paper draws from the heterogeneous-agent asset pricing literature. We model an economy populated by agents with different levels of risk aversion, which gives rise to a credit market as in Dumas (1989), Wang (1996), Gârleanu and Panageas (2008), and Longstaff and Wang (2012). Our model is also related to the literature on collateral or margin constraints and their effects on asset prices (e.g. Gromb and Vayanos, 2002; Geanakoplos, 2003; Brunnermeier and Pedersen, 2009; Gârleanu and Pedersen, 2011; Ashcraft, Garleanu, and Pedersen, 2011; Gorton and Ordoñez, 2014; Moreira and Savov, 2014). In our model the tightness of the leverage constraint depends on the nominal interest rate, which gives monetary policy the power to influence leverage in the financial sector.

Importantly for our mechanism, the nominal interest rate must influence the premium investors pay for holding liquid assets, as in Figure 1. Nagel (2014) shows that there is in fact a tight positive relationship between the nominal rate and the liquidity premium for a wide variety of liquid ("near-money") assets within and across a number of countries and over a long sample period. Drechsler, Savov, and Schnabl (2014) show that an increase in the nominal rate causes an increase in the premium for holding retail bank deposits, an important source of liquidity in the economy. Thus, the liquidity premium mechanism at the heart of the model finds strong support in the data.

A large empirical literature documents a strong influence of monetary policy on credit supply (Bernanke and Blinder, 1992; Kashyap, Stein, and Wilcox, 1993; Bernanke and Gertler, 1995; Kashyap and Stein, 2000). Recent papers employ micro-level data to rule out confounding factors (Jiménez, Ongena, Peydró, and Saurina, 2014; Dell'Ariccia, Laeven, and Suarez, 2013; Landier, Sraer, and Thesmar, 2013; Scharfstein and Sunderam, 2014; Drechsler, Savov, and Schnabl, 2014). Broadly speaking, these papers find that nominal rate increases cause banks to reduce deposits, lending, and risk taking.

A central prediction of our model is that low nominal interest rates result in low risk premia, which is sometimes called "reaching for yield". A fast-growing literature finds support for this relationship. Bernanke and Kuttner (2005) show that surprise rate increases induce large negative stock returns. Using a vector autoregression, they find this result is almost entirely driven by higher expected excess returns, consistent with our model. Hanson and Stein (2014), Gertler and Karadi (2014), and Bekaert, Hoerova, and Lo Duca (2013) document parallel results for term premia, credit spreads, and the pricing of volatility risk. Bekaert, Hoerova, and Lo Duca (2013) find in particular that an increase in the nominal rate raises the representative investor's risk aversion, which is precisely the channel of our model.

Finally, our paper is related to the literature on the role of safe assets in financial markets (Lucas, 1990; Woodford, 1990; Gertler and Karadi, 2011; Caballero and Farhi, 2013; Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood, Hanson, and Stein, 2013). By providing liquidity to banks, government bonds in our model can "crowd in" investment and risk taking.

III. Model

In this section we lay out our model. The setting is an infinite-horizon economy that evolves in continuous time $t \geq 0$. For simplicity, we begin with an endowment framework, leaving production for Section VI below.

A. Endowment and agents

The aggregate endowment follows a geometric Brownian motion:

$$\frac{dY_t}{Y_t} = \mu_Y dt + \sigma_Y dB_t. \tag{1}$$

The economy is populated by a continuum of agents whose total mass is one. There are two types of agents, A and B. Both types have recursive preferences as in Duffie and Epstein (1992), the continuous-time analog of Epstein and Zin (1989). These preferences allow us to separate the elasticity of intertemporal substitution (EIS) from the coefficient of relative risk aversion. An EIS greater than one ensures that discount rates are increasing in risk aversion so that higher risk premia result in lower asset prices and a higher cost of capital.

To ensure stationarity, we assume that agents die at a rate κ. New agents are also born at a rate κ with a fraction $\overline{\omega}$ as type A and $1-\overline{\omega}$ as type B. The newly-born inherit the wealth of the deceased on an equal per-capital basis. Gârleanu and Panageas (2008) show that under these conditions κ simply increases agents' effective rate of time preference. Hence the lifetime utility V_0^i of an agent of type $i \in \{A, B\}$ is given by the recursion

$$V_0^i = E_0 \left[\int_0^\infty f^i \left(C_t^i, V_t^i \right) dt \right] \tag{2}$$

$$f^i \left(C_t^i, V_t^i \right) = \left(\frac{1 - \gamma^i}{1 - 1/\psi} \right) V_t^i \left[\left(\frac{C_t^i}{\left[\left(1 - \gamma^i \right) V_t^i \right]^{1/(1-\gamma^i)}} \right)^{1-1/\psi} - (\rho + \kappa) \right]. \tag{3}$$

The felicity function f^i is an aggregator over current consumption C_t^i and future utility V_t^i. The parameters ρ and ψ, which we take as common for simplicity, denote agents' subjective discount rate and elasticity of intertemporal substitution (EIS).

The essential source of heterogeneity in the model lies in the relative risk aversion coefficients γ^i. Without loss of generality, A agents are less risk averse than B agents: $\gamma^A < \gamma^B$. It follows that in equilibrium A agents raise funds from B agents and use them to make levered investments. We think of the combined net worth of A agents as the pooled equity capital of financial intermediaries. For brevity we refer to these simply as banks, but they can be thought of more broadly to include not just commercial banks but also off-balance sheet vehicles, broker-dealers and hedge funds. The unifying trait of these institutions is that they lever up their equity using short-term debt financing.[7]

Along the same lines, we refer to the liabilities of A agents to B agents as deposits. As we discuss below, we interpret deposits as encompassing the many types of short-term debt financing used by financial institutions, including retail deposits and wholesale deposits (i.e. wholesale funding) such as large CDs and commercial paper.

Let W_t^i be the combined wealth of type-i agents, and let ω_t be the wealth share of A agents,

$$\omega_t = \frac{W_t^A}{W_t^A + W_t^B}. \tag{4}$$

As we show below, ω_t summarizes the state of the economy for a given level of output. Its law of motion can be expressed as

$$d\omega_t = \kappa \left(\overline{\omega} - \omega_t \right) dt + \omega_t \left(1 - \omega_t \right) \Big[\mu_\omega \left(\omega_t \right) dt + \sigma_\omega \left(\omega_t \right) dB_t \Big]. \tag{5}$$

[7]Our setting can also be applied to open-end mutual funds, including money market funds and long-term equity and bond funds. Mutual funds issue shares that are redeemable daily at net asset value (NAV), which creates funding risk that can lead to fire sales (Coval and Stafford, 2007). A prominent example occurred in September 2008 when money market funds experienced a run (Kacperczyk and Schnabl, 2013). Because their liquid assets were insufficient to meet redemptions, the U.S. Treasury was compelled to issue them a blanket guarantee to prevent them from "breaking the buck". Similarly, to protect against sudden withdrawals long-term equity and bond funds on average hold 5% of total assets in cash, ranging from 1% for government bond funds to 15% for global bond funds (Investment Company Institute, 2014).

It has an exogenous component due to demographics that ensures a non-degenerate long-run distribution, and an endogenous component (in brackets) due to differences in saving rates and portfolio choices across the two types of agents.

All agents can trade a claim on the aggregate endowment, which we call the risky asset. The price of this claim is P_t, its dividend yield is $F(\omega_t) = Y_t/P_t$, and its return process is

$$dR_t = \frac{dP_t + Y_t dt}{P_t} = \mu_t dt + \sigma_t dB_t. \tag{6}$$

We solve for the expected return $\mu_t = \mu(\omega_t)$ and volatility $\sigma_t = \sigma(\omega_t)$ in equilibrium.

Agents also trade instantaneous risk-free bonds, i.e. deposits, amongst each other. Deposits pay the endogenously-determined real interest rate $r_t = r(\omega_t)$. The other securities in this economy are government bonds and reserves, which we describe below.

B. The demand for liquid assets

The different risk tolerances of A and B agents create gains from risk sharing. In equilibrium, A agents raise deposits from B agents and use the funds to take a levered position in the risky asset. However, in doing so they are confronted with a friction.

The friction is that deposits are subject to funding shocks. We model them as arriving according to a Poisson process N_t with constant intensity η. Funding shocks affect all A agents (banks) at once, i.e. they are systematic in nature and cannot be diversified away within the banking system.[8] Consider an agent who has raised deposit funding. If hit with a funding shock, the agent must immediately redeem a fraction

$$\frac{\lambda}{1+\lambda} \tag{7}$$

of her deposits, where $\lambda > 0$. Funding shocks are important because having to rapidly liquidate risky assets results in a fire sale. Specifically, only a fraction $1 - \phi \in (0,1)$ of the value of a risky asset can be recovered quickly enough to absorb a funding shock. Fire sales can arise due to adverse selection or limited expertise leading to "cash-in-the-market" pricing (Allen and Gale, 1994). These problems are avoided when assets are sold over a longer period but not on such short notice.

The combination of funding shocks and fire sales plays an important role in the banking literature (Diamond and Dybvig, 1983; Bhattacharya and Gale, 1987; Shleifer and Vishny, 1997; Holmström and Tirole, 1998). In this literature, financial intermediaries issue short-

[8]To clarify, we do not model idiosyncratic or bank-specific shocks since these can in principle be shared perfectly in interbank markets and should therefore not command a premium. Equivalently, we can view our representative A banks as having already perfectly shared all idiosyncratic risks.

term liabilities and use them to invest in risky illiquid assets. Households demand these liabilities because of their safety and liquidity, but supplying them exposes financial intermediaries to rollover risk. Our funding shocks capture this notion.[9]

Agents can self-insure by holding assets that are immune to fire sales and can therefore be liquidated at low cost in the event of a funding shock. We call these liquid assets. By holding enough liquid assets, banks can avoid having to sell risky assets at fire sale prices in case of a funding shock. By influencing the liquidity premium, which is the opportunity cost of holding liquid assets, monetary policy affects the cost of this self-insurance and hence banks' demand for risk taking.

Note that because funding shocks require the banking system as a whole to shrink its liabilities, interbank claims cannot serve as liquid assets. Rather, the liquid assets held by banks must be claims on entities that lie outside the banking system, such as the government. This point echoes the broader argument in Holmström and Tirole (1998), who emphasize the special role of public liquidity provision.

Policy makers have embraced the principle of holding liquid asset buffers as insurance against funding risk (Stein, 2013). Regulations requiring liquid asset buffers include reserve requirements and the liquidity coverage and net stable funding ratios adopted by Basel III and U.S. regulators (Basel Committee on Banking Supervision, 2013, 2014; Board of Governors of the Federal Reserve System, 2014). These rules are meant to ensure that banks avoid forced liquidations, a rationale in keeping with our framework. Banks in our model self-insure voluntarily but the implications remain the same if they do so to comply with regulation.

C. The supply of liquid assets

We model two types of liquid assets. They are in finite supply and cannot be sold short by agents. The first is instantaneous risk-free government bonds.[10] The important feature of government bonds is that they trade in markets that remain highly liquid even at times of severe market stress. This feature is shared not only by Treasury bonds but also by agency-backed MBS and other implicitly or explicitly guaranteed instruments. Hence we refer to all of these as government bonds. We model government bonds as providing liquidity services normalized to one.

[9] Recently, rollover risk played a important role in the 2008 financial crisis (Acharya, Gale, and Yorulmazer, 2011). Since the crisis, financial institutions have responded in part by stocking greater quantities of liquid assets (Financial Stability Oversight Council, 2014), consistent with our model.

[10] In practice government bonds have non-zero maturity and interest rate risk exposure. A way to map the liquidity value of long term bonds into our model is to measure it as the amount that can be borrowed against them in a repo transaction. Because government bonds are safe and liquid, their repo haircuts are small, so this amount is close to their full market value. Hence, making such an adjustment would not significantly affect the interpretation of our model.

The second liquid asset is central bank reserves (including currency), which are long-lived. Banks trade reserves in large volumes in the federal funds market (Afonso, Kovner, and Schoar, 2011). They are an even more efficient source of liquidity than government bonds because of their fixed nominal value and ability to circulate more quickly within the banking system. We therefore model reserves as having a liquidity multiplier: each dollar of reserves provides $m > 1$ liquidity services.

The central bank creates and withdraws reserves by exchanging them for government bonds, i.e. via open market operations. This has the effect of expanding and contracting the outstanding supply of liquidity. Let G_t and M_t be the dollar value of government bonds and reserves in the economy and let π_t be the value of a dollar in consumption units. Taking reserves as the numeraire (in practice they are fungible with currency so this is the natural choice), π_t is the inverse price level. Then the real value of liquidity held by the public, measured in units of government bonds, and scaled by the value of the endowment P_t is

$$\Pi_t = \frac{\pi_t \left[G_t + (m-1) M_t\right]}{P_t}. \tag{8}$$

As a result of open market operations, the remaining liquidity, which is given by the value of government bonds held by the central bank, $\pi_t M_t$, is held by the central bank itself.

D. *Inflation and the nominal rate*

We restrict attention to policies under which inflation is locally deterministic

$$-\frac{d\pi_t}{\pi_t} = \iota_t dt. \tag{9}$$

Besides being realistic, this restriction has the virtue of simplifying the analysis without limiting the central bank's ability to influence the economy. We discuss this in more detail below when we show how (9) is achieved. Note that (9) implies that deflation $-\iota_t$ is the capital gain on reserves, and that reserves are locally risk-free.

The nominal interest rate is equal to the real rate plus inflation:

$$n_t = r_t + \iota_t. \tag{10}$$

In practice n_t corresponds to the fed funds rate. The fed funds market is one of two main sources of overnight unsecured funding for U.S. banks.[11] The fed funds rate therefore equals the rate banks pay on a marginal dollar of funding. In our model, banks are funded entirely

[11] The other is the Eurodollar market whose prevailing rate, LIBOR, aligns closely with the fed funds rate (Kuo, Skeie, and Vickery, 2012).

with deposits and hence the fed funds rate and the deposit rate are the same.

In reality banks issue a variety of deposits that pay different rates. There are two broad classes, wholesale and retail deposits. Wholesale deposits include large CDs, commercial paper, fed funds, and Eurodollars. Their rates are close to the fed funds rate and they map to the model's notion of deposits in a straight-forward way.

Retail deposits, on the other hand, are sold in a setting where banks have market power and as a result pay substantially lower rates (Drechsler, Savov, and Schnabl, 2014). They also involve significant non-interest (i.e. brick-and-mortar) costs. As a result, the true marginal cost of retail deposits for banks is higher than the rate they pay their depositors. Indeed, for banks to be indifferent between their alternative sources of funding, the marginal cost of retail deposits must also equal the fed funds rate. For the purposes of our model we therefore group retail and wholesale deposits into a single deposit category.

The fed funds rate has emerged as the key target of monetary policy in the U.S.. Likewise, we take

$$n_t = n(\omega_t) \tag{11}$$

to be the central bank's target and show how it gets implemented. The rule that specifies the evolution of the target is a function of ω_t, which summarizes the state of the economy. We assume agents have rational expectations and therefore know this mapping. In Section VI, we extend the model by introducing unexpected shocks to the policy rule.

Turning to the supply of government bonds, for simplicity we assume that it evolves according to an exogenous function of the state ω_t, and that the Treasury issues and redeems government bonds in exchange for deposits. This assumption prevents Treasury issuance from having redistributive effects. Moreover, as we show in Section IV.B below, equilibrium in our model does not depend on the precise path of Treasury issuance because the central bank can respond as required to achieve its nominal rate target.

E. *The liquidity premium*

The liquidity premium in our model depends directly on the nominal rate n_t. To understand this, consider the case of reserves. In equilibrium the liquidity premium on reserves must equal the opportunity cost of holding them in terms of foregone return. As reserves pay no interest, their return is equal to their capital gain $d\pi_t/\pi_t$. Hence the opportunity cost of reserves, as well as their liquidity premium, is equal to

$$r_t - \frac{d\pi_t}{\pi_t} = r_t + \iota_t = n_t. \tag{12}$$

In other words, the nominal interest *is* the liquidity premium on reserves.[12]

Let the real interest rate on government bonds be $r^g_t = r^g(\omega_t)$. Since government bonds provide liquidity services that are $1/m$ those of reserves, their liquidity premium is

$$r_t - r^g_t \;=\; \frac{1}{m} n_t. \tag{13}$$

Thus the liquidity premium on government bonds moves with the nominal rate. This relationship would also extend to other liquid assets.

Figure 1 shows that this relationship holds empirically. It plots the fed funds rate (solid red line, left axis) against the spread between fed funds and the 3-month T-bill (black squares, right axis) from January 1955 to December 2009. The two series co-move very strongly both in the trend and in the cycle; their raw correlation is 78%.[13] Nagel (2014) documents a similar relationship over a long time period, across a number of liquid assets, and in several countries. These results support the transmission mechanism underlying our model, the tight relationship between the level of nominal interest rates and liquidity premia.

In conducting monetary and fiscal policy, the government earns "seigniorage" profits due to the differential liquidity of its assets and liabilities. Scaled by the value of the endowment, this seigniorage accrues at a rate

$$\frac{\pi_t G_t}{P_t}\left(r_t - r^g_t\right) + \frac{\pi_t M_t}{P_t}\left(r^g_t - \frac{d\pi_t}{\pi_t}\right) \;=\; \frac{\Pi_t}{m} n_t. \tag{14}$$

The terms on the left are the Treasury and central bank accounts. The equality follows by substituting (12) and (13) for the liquidity premia and simplifying using (8). We see that seigniorage equals the liquidity premium on government bonds, n_t/m, times the total liquidity held by the public, Π_t, itself measured in government bonds. We also see that seigniorage is non-negative and indeed positive so long as the nominal rate is above zero. To close the model, we assume that as seigniorage accrues it is refunded to all agents in proportion to their wealth. This assumption leaves the wealth distribution unaffected and keeps the government's net worth at zero.

[12] How would paying interest on reserves affect this relationship? Let ior_t be the interest paid on reserves. The foregone return in (12) becomes $n_t - ior_t$ which, all else equal, decreases with interest on reserves. The central bank can now target the *difference* $n_t - ior_t$ to achieve a desired liquidity premium just as it does absent interest on reserves. Hence, by itself paying interest on reserves does not limit the central bank's ability to target the liquidity premium. Section IV.B discusses the impact of interest on reserves on the implementation of monetary policy.

[13] The occasional spikes in the fed funds-T-bill spread coincide with "flight to quality" episodes in which the T-bill rate plummets. The spread briefly widens because many non-banks (e.g. money market funds) cannot access reserves or invest in long-term government bonds. From the perspective of our model, this can be understood as a temporary breakdown of the infinite substitutability of government bonds and reserves.

F. Agents' optimization problem

In this section we write down the optimization problem of an agent in our economy. Let $V^i(W_t, \omega_t)$ denote the value function of an agent of type $i \in \{A, B\}$. The agent chooses a consumption-wealth ratio c_t and portfolio shares $w_{S,t}$, $w_{D,t}$, $w_{G,t}$, and $w_{M,t}$ in the risky asset, deposits, government bonds, and reserves (these sum to one).

To simplify the portfolio problem, we use the fact that in equilibrium reserves and bonds are perfect substitutes in the agent's portfolio and express her optimization problem in terms of an overall demand for liquidity given by $w_{L,t} = w_{G,t} + m w_{M,t}$, in units of government bonds. The resulting Hamilton-Jacobi-Bellman (HJB) equation is

$$0 = \max_{c_t, w_{S,t}, w_{L,t} \geq 0} f^i\left(c_t W_t, V^i(W_t, \omega_t)\right) dt + E_t\left[dV^i(W_t, \omega_t)\right], \tag{15}$$

subject to the wealth dynamics[14]

$$\begin{aligned} \frac{dW_t}{W_t} &= \left[r_t - c_t + w_{S,t}(\mu_t - r_t) - \frac{w_{L,t}}{m} n_t + \frac{\Pi_t}{m} n_t\right] dt + w_{S,t}\sigma_t dB_t \\ &\quad - \frac{\phi}{1-\phi} \max\left\{\frac{\lambda}{1+\lambda}(w_{S,t} + w_{L,t} - 1) - w_{L,t}, 0\right\} dN_t. \end{aligned} \tag{16}$$

(Appendix A contains the full derivation.) The drift component of wealth reads as follows: the agent earns the deposit rate, pays for consumption, earns the risk premium on the risky asset, pays the liquidity premium on her liquid holdings, and receives seigniorage payments from the government.[15] Liquidity demand expressed in units of effective reserves is $w_{L,t}/m$ and the associated excess return from (12) is $-n_t$. Hence, holding liquidity is costlier when the nominal rate is high.

The diffusive component of wealth in (16) only depends on exposure to the risky asset. This is because all other assets are locally risk-free, including reserves (see (9)).

The second line of (16) captures the agent's exposure to the funding shock dN_t. Inside the braces is the amount of redemptions in excess of the available liquid holdings. If there is enough liquidity to cover all redemptions, the funding shock exposure is zero. If not, the agent must sell enough risky assets to cover the shortfall. Because of the fire sale, it takes $1/(1-\phi)$ dollars of the risky asset to meet one dollar of redemptions, and each dollar sold incurs ϕ dollars of fire sale losses. Below we derive conditions under which it is optimal for banks to fully self-insure so these fire sale losses are zero in equilibrium.

[14] These are the wealth dynamics should the agent cheat death over the next instant. The agent accounts for the possibility of death directly in the felicity function (3).

[15] From (14), total seigniorage is $P_t \Pi_t n_t / m$. Since seigniorage is refunded in proportion to wealth and since total wealth is P_t, each agent gets $\Pi_t n_t / m$ in seigniorage per unit of wealth.

G. *Market clearing conditions*

The homogeneity of preferences implies that the consumption and portfolio policies are independent of wealth, so we can write them as functions of type. We denote the aggregated consumption-wealth ratio of type i agents by $c_t^i = c^i(\omega_t) = \int_i c_t^h W_t^h / W_t^i dh$ for $i \in \{A, B\}$, and similarly for the portfolio policies $w_{S,t}^i = w_S^i(\omega_t)$ and $w_{L,t}^i = w_L^i(\omega_t)$.

In equilibrium, the markets for goods (i.e. consumption), the endowment claim, government bonds, and reserves must clear. The deposit market clears by Walras' law. Since government bonds and reserves are perfect substitutes, only the total demand for liquidity is pinned down and the two markets can be consolidated. Since the public's net deposit holdings are minus the value of government bonds and reserves, aggregate wealth equals the value of the endowment claim, $W_t^A + W_t^B = P_t$. The market clearing conditions are thus

$$\begin{aligned} \omega_t c_t^A + (1-\omega_t) c_t^B &= F_t && (17)\\ \omega_t w_{S,t}^A + (1-\omega_t) w_{S,t}^B &= 1 && (18)\\ \omega_t w_{L,t}^A + (1-\omega_t) w_{L,t}^B &= \Pi_t. && (19)\end{aligned}$$

All three conditions are normalized by aggregate wealth. The first equation gives the goods-market clearing condition, the second gives the market clearing condition for the endowment claim, and the third gives the market clearing condition for the liquid assets.

IV. Equilibrium

In this section we derive the equations that characterize the equilibrium. While these equations do not permit closed-form solutions, we are able to derive analytical expressions that highlight key mechanisms. In Section V, we solve the model fully using numerical methods.

All proofs and derivations are in Appendix A.

A. *The value function and the demand for leverage*

To simplify notation we now drop time subscripts, though it should be understood that they apply. By Ito's Lemma, the HJB equation (15) expands into the Lagrangian

$$\begin{aligned} 0 = & \max_{c, w_S, w_L \geq 0} f^i\left(cW, V^i\right) + V_W^i W \left[r - c + w_S(\mu - r) - \frac{w_L}{m} n + \frac{\Pi}{m} n\right] \qquad (20)\\ & + V_\omega^i \Big[\kappa(\overline{\omega} - \omega) + \omega(1-\omega)\mu_\omega\Big] + V_{W\omega}^i W \omega (1-\omega) w_S \sigma_\omega \sigma + \frac{1}{2} V_{WW}^i W^2 (w_S \sigma)^2 \\ & + \frac{1}{2} V_{\omega\omega}^i \omega^2 (1-\omega)^2 \sigma_\omega^2 + \eta\left(V_+^i - V^i\right), \end{aligned}$$

where $V_{+}^{i}=V^{i}\left(W_{+},\omega\right)$ and W_{+} is the agent's wealth immediately following a funding shock. Since the funding shock can only destroy wealth, $W_{+}-W\leq 0$ with equality when the agent has sufficient liquidity to insure against the funding shock.

This highlights the banks' tradeoff: holding liquidity requires paying a premium but it also reduces the negative impact of a funding shock, $V_{+}^{i}-V^{i}$. It is clear that as long as liquidity is costly, banks will never hold more liquidity than is necessary to fully self-insure. For simplicity, from now on we restrict the parameters so that banks in fact choose to fully self-insure:

Assumption. *Let η and ϕ be such that for all $n\left(\omega\right)$,*

$$\eta\frac{\phi}{1-\phi}\frac{\lambda}{1+\lambda} \quad \geq \quad \frac{\lambda}{m}n\left(\omega\right). \tag{21}$$

For full self-insurance to be optimal, expected fire sale losses (the left side of (21)) must be high enough to justify paying the liquidity premium on the amount of liquidity required to insure a dollar of deposits, λ. We verify this condition holds for our chosen parameters (see Appendix A). Under full insurance, banks' liquidity demand is

$$w_{L}^{A} \quad = \quad \max\left\{\lambda\left(w_{S}^{A}-1\right),0\right\} \tag{22}$$

for a given level of risky asset demand w_{S}^{A}. Equation (22) shows that liquidity demand is proportional to net leverage. This is the key mechanism that connects liquidity and risk taking in our model.[16]

We can now turn to the full optimization problem and characterize the value function:

Proposition 1. *The value function of an agent of type $i\in\{A,B\}$ has the form*

$$V^{i}\left(W,\omega\right) \quad = \quad \left(\frac{W^{1-\gamma}}{1-\gamma}\right)J^{i}\left(\omega\right)^{\frac{1-\gamma}{1-\psi}}, \tag{23}$$

where J^{i} is the agent's consumption-wealth ratio, $c=J^{i}$. If $\left(\lambda/m\right)n\leq\left(\gamma^{B}-\gamma^{A}\right)\sigma_{Y}^{2}$, J^{i}

[16] We stress that full insurance is not required for liquidity demand to increase with leverage. Rather, it is enough that banks insure themselves up to some probability. In practice, banks do hold large liquidity buffers (see the discussion in Section V.A below). Bai et al. (2013) find that the largest banks typically hold enough liquid assets to withstand a 3-σ liquidity stress test.

solves the second-order ordinary differential equation

$$
\begin{aligned}
\rho+\kappa \;&=\; 1/\psi J^{i}+(1-1/\psi)\left(r+\lambda\theta^{i}+\frac{\Pi}{m}n\right)-1/\psi\frac{J_{\omega}^{i}}{J^{i}}\Big[\kappa\left(\overline{\omega}-\omega\right) \\
&\quad +\omega\left(1-\omega\right)\mu_{\omega}\Big]-\frac{1/\psi}{2}\left[\left(\frac{\psi-\gamma^{i}}{1-\psi}\right)\left(\frac{J_{\omega}^{i}}{J^{i}}\right)^{2}+\frac{J_{\omega\omega}^{i}}{J^{i}}\right]\omega^{2}\left(1-\omega\right)^{2}\sigma_{\omega}^{2} \\
&\quad +\frac{1}{2}\left(\frac{1-1/\psi}{\gamma^{i}}\right)\left[\frac{\mu-r-\lambda\theta^{i}}{\sigma^{2}}+\left(\frac{1-\gamma^{i}}{1-\psi}\right)\frac{J_{\omega}^{i}}{J^{i}}\omega\left(1-\omega\right)\frac{\sigma_{\omega}}{\sigma}\right]^{2}\sigma^{2}
\end{aligned}
\tag{24}
$$

with $\theta^{A}=n/m$ and $\theta^{B}=0$. If instead $\left(\lambda/m\right)n>\left(\gamma^{B}-\gamma^{A}\right)\sigma_{Y}^{2}$, J^{i} solves

$$
\rho+\kappa \;=\; 1/\psi J^{i}+(1-1/\psi)\left(\mu-\frac{\gamma^{i}}{2}\sigma_{Y}^{2}\right)-1/\psi\frac{J_{\omega}^{i}}{J^{i}}\Big[\kappa\left(\overline{\omega}-\omega\right)+\omega\left(1-\omega\right)\mu_{\omega}\Big]. \tag{25}
$$

Proposition 1 shows that the value function is homogeneous in wealth. It can be characterized up to the consumption-wealth ratio J^{i}, which is type-specific but not agent-specific and depends solely on the wealth distribution ω. As a result, ω is a sufficient statistic for asset valuations and other equilibrium quantities in this economy.

Using the value functions, we can solve for agents' portfolio demands. Proposition 2 below provides the conditions under which A agents (banks) take leverage (by issuing deposits), and shows how their demand for the risky asset depends on monetary policy as captured by the nominal interest rate.

Proposition 2. *Banks take leverage/deposits ($w_{S}^{A}>1$) if and only if*

$$
\frac{\lambda}{m}n \;<\; \left(\gamma^{B}-\gamma^{A}\right)\sigma_{Y}^{2}. \tag{26}
$$

In this case, their demand for the risky asset is given by

$$
w_{S}^{A} \;=\; \frac{1}{\gamma^{A}}\left[\frac{\mu-r-\left(\lambda/m\right)n}{\sigma^{2}}+\left(\frac{1-\gamma^{A}}{1-\psi}\right)\frac{J_{\omega}^{A}}{J^{A}}\omega\left(1-\omega\right)\frac{\sigma_{\omega}}{\sigma}\right]. \tag{27}
$$

Equation (27), which describes banks' leverage, has two terms: In the first, $(\mu-r)/\sigma^{2}$ is the canonical mean-variance tradeoff for the risky asset. It shows that banks take more leverage when there is a higher return premium per unit of risk. The last term, which depends on J_{ω}^{A}, captures the intertemporal hedging demand. It determines how much banks adjust their current risk taking to hedge future changes in investment opportunities. The investment opportunity set is stochastic because of variation in aggregate risk aversion that is induced by changes in the relative wealth ω of the two types of agents. The term $-\left(\lambda/m\right)n$ in (27) gives the direct impact of the nominal rate on bank leverage, which we highlight with the following corollary:

Corollary 1. *All else equal, an increase in the nominal interest rate reduces bank leverage.*

For each additional dollar of leverage, banks must expand their liquid holdings to avoid fire sales. Doing so is costly because these assets carry a liquidity premium. The liquidity premium is proportional to the nominal rate. As a result, higher nominal rates raise the effective cost of leverage.[17]

Looking at (27), an increase in the nominal rate works like an increase in banks' effective risk aversion. As a result, an increase in the nominal rate raises the economy's effective aggregate risk aversion and in equilibrium, the risk premium.

Condition (26) shows that banks lever up only if there is enough scope for risk sharing to overcome the cost of leverage. The difference in risk aversions multiplied by the return variance, $\left(\gamma^B - \gamma^A\right)\sigma_Y^2$, measures the risk premium earned by banks on their first dollar of leverage.[18] This premium reflects the gains from risk sharing. For banks to take leverage, it must be greater than the cost of leverage, which is given by the liquidity premium multiplied by the required liquidity buffer, $(\lambda/m)\,n$.

On the other hand, if the cost of leverage exceeds the benefit of sharing risk, even on the first dollar, then banks do not raise deposits and agents remain in "financial autarky":

Corollary 2. *If* $(\lambda/m)\,n \geq \left(\gamma^B - \gamma^A\right)\sigma_Y^2$ *then* $w_S^A = w_S^B = 1$.

The demand for leverage equation (27) shows that banks in effect discount assets using a risk-adjusted rate $r + (\lambda/m)\,n$ that is higher than the rate they pay on deposits, r. The difference, $(\lambda/m)\,n$, is the cost of holding liquidity as insurance against funding shocks. This difference drives a wedge between the cost of external funds for firms, the ultimate borrowers in the economy, and the return earned by households, the ultimate lenders. The literature (e.g. Bernanke and Gertler, 1995) calls this wedge the external finance premium and shows that it increases with the nominal rate, as implied by our model.

B. Policy implementation

We now discuss in more detail how the central bank influences the nominal interest rate in the model. We begin with the following proposition which characterizes the dynamics of

[17]The nominal interest rate thus acts as an tax on leverage. Our model thus highlights the connection between monetary policy and macro-prudential regulation. Stein (2012) argues that monetary policy has several advantages that complement a regulation-based approach such as an explicit tax. First, monetary policy permeates the entire financial system, affecting every institution that takes leverage implicitly or explicitly, thus making it less susceptible to distortions such as regulatory arbitrage and hidden leverage. Second, monetary policy (the nominal rate) can be adjusted dynamically in response to changing economic conditions. Third, the nominal rate and liquidity premia represent a useful price-based signal of policy tightness that can guide policy makers.

[18]In the no-leverage region return volatility equals fundamental volatility, $\sigma = \sigma_Y$.

aggregate liquidity required to implement a given policy $n_t = n(\omega_t)$:

Proposition 3. *To implement the nominal rate rule $n_t = n(\omega_t)$, the nominal supply of reserves M_t and government bonds G_t must grow according to*

$$\alpha_t \frac{dM_t}{M_t} + (1-\alpha_t)\frac{dG_t}{G_t} = (n_t - r_t)\,dt + \frac{dP_t}{P_t} + \frac{d\Pi_t}{\Pi_t} + \left(\frac{dP_t}{P_t}\right)\left(\frac{d\Pi_t}{\Pi_t}\right), \tag{28}$$

where $\alpha_t = (m-1)\,\pi_t M_t / (\Pi_t P_t)$ is the net contribution of reserves to aggregate liquidity and Π_t, the real value of liquidity as a share of wealth, is given by $\Pi_t = \omega_t \lambda (w_{S,t}^A - 1)$.

Proposition 3 says that in order to implement a given nominal interest rate, the supply of liquid assets must evolve according to (28). The evolution of the supply of liquid assets is a weighted average of the changes in the supply of reserves and government bonds because these two sources of liquidity are perfectly substitutable. Hence, it is their combined supply that matters for policy implementation.

To understand (28), first consider holding government bond issuance fixed. To achieve a high nominal rate through open market operations, (28) shows that the central bank must issue reserves at a high rate going forward. Doing so causes reserves to decline in value more quickly over time (i.e. inflation rises), and hence increases the opportunity cost of holding them. As explained in Section III.E, this opportunity cost is the nominal rate.[19] When the nominal rate rises, the liquidity premium of government bonds also rises. This leads banks to reduce their demand for liquidity today, the real value of aggregate liquidity falls ($d\Pi_t/\Pi_t$ is low), and this is absorbed by open market operations today (dM_t/M_t is low). The right side of (28) further shows that the liquidity supply must keep up with overall wealth in the economy (dP_t/P_t).

Proposition 3 also shows that a given nominal rate and the liquidity premium it implies can be implemented through changes in the supply of liquid assets other than reserves (dG_t/G_t). While we have labeled these government bonds, in practice many liquid assets are originated by the private sector. The required adjustment in the liquidity supply can occur through these assets, with only a minimal need for open market operations. Indeed, the evidence in Drechsler, Savov, and Schnabl (2014) shows that large flows of retail bank deposits play such a role. They show that when the nominal rate increases, the premium for holding retail deposits, a large and important class of liquid assets, increases sharply, and their quantity shrinks. They argue that this is due to pricing power in the retail deposit market, which increases with the nominal rate.[20] Although we do not explicitly incorporate

[19]Introducing price rigidities into our model would dampen the change in reserves growth required to implement a change in the nominal rate as an increase in the nominal rate would result in a similar increase in the real rate (i.e. the change in $n_t - r_t$ would be small).

[20]The mechanism for the increase in pricing power is as follows: When the nominal rate is low, retail

this particular channel, it can be subsumed into the process dG_t/G_t, which would then encompass the change in both government bonds and other liquid securities. Regardless of whether the change in the liquidity supply occurs through reserves (dM_t/M_t) or other liquid assets (dG_t/G_t), the end result is that by changing the nominal rate the central bank changes the liquidity premium. Given this effect, which is clearly visible in Figure 1, the model's implications follow.

As we noted earlier, with interest on reserves (IOR), the opportunity cost of holding reserves equals $n_t - ior_t$. In this case banks' demand for leverage $w^A_{S,t} - 1$ is given by (27) but with $n_t - ior_t$ replacing n_t. Proposition 3 is then adapted to the case with IOR by substituting this expression in for $w^A_{S,t} - 1$. This shows that when the spread $n_t - ior_t$ is low, banks' demand for liquid assets is high similar to the case when n_t is low, and the real value of liquidity as a share of wealth Π_t is high. Thus, by paying high IOR, the central bank can keep outstanding liquid holdings high even as it raises the nominal rate. It may be interested in doing so if raising the nominal rate (though without changing the liquidity premium) helps to maintain price stability.[21]

V. Results

In this section we further analyze the effects of monetary policy by setting parameter values, specifying a nominal rate policy, and solving for the resulting equilibrium. This requires solving the HJB equations of the two types of agents simultaneously. Since these do not permit closed-form solutions, we apply numerical methods, specifically Chebyshev collocation, which provides a global solution.

A. Parameter values

Table I displays our benchmark parameter values. While our main goal is to illustrate the mechanisms of the model, we pick parameter values that we view as reasonable.

In order to create substantial demand for risk sharing and leverage, we set the risk aversions of A and B agents to 1.5 and 15. We set the common elasticity of intertemporal

deposits must compete with cash and non-interest accounts, whereas when it is high, banks are able to charge a bigger liquidity premium (pay relatively lower rates). The increased liquidity premium leads to outflows and a reduction in the supply of liquid assets in the economy.

[21] We note the following caveat. Segmentation frictions can lead to an imperfect pass-through of IOR to the marginal cost of liquidity in the financial system (see Bech and Klee, 2011, for an example from the post-2008 period). Despite these frictions, when reserve balances are large the level of IOR is likely to have a strong influence on the marginal cost of liquidity. This may no longer hold when reserves are small. Still, as long as the supply of other liquid assets (such as retail bank deposits) responds to changes in the nominal rate, the nominal rate will influence the liquidity premium.

substitution (EIS) to 3.[22] An EIS greater than one implies that an increase in effective risk aversion decreases valuations.

We set agents' rate of time preference ρ and death rate κ to 0.01, which results in real interest rates near 2%. To stabilize banks' share of wealth ω_t at moderate levels, we set the fraction of A agents $\overline{\omega}$ to 0.1. We set the growth rate and volatility of the endowment to 2%, consistent with standard estimates for aggregate U.S. consumption.

Next we set λ, which governs the size of funding shocks (see (7)). There are several approaches we can take. One approach is to look at the recent 2008 financial crisis which saw a dramatic contraction in wholesale funding markets. Acharya et al. (2013) show that from July to December 2007, the outstanding amount of asset-backed commercial paper contracted by 36%. Krishnamurthy et al. (2014) calculate that from 2007 to 2009, the overall amount of short-term debt used to fund purchases of risky private-label ABS and corporate bonds contracted by 66% and 29% respectively, and by 57% overall.

As the recent crisis may have been particularly severe, a more reliable approach may be to look at the amount of liquid assets banks hold on their balance sheets. Intuitively, the amount of insurance banks buy informs us about the funding risk they perceive. In 2004, 2008, and 2013, U.S. commercial banks held 18%, 21%, and 25% of assets in liquid securities and had 73%, 76%, and 77% of liabilities in deposits and other short-term debt. Hence banks' liquid securities as a proportion of deposits were 25%, 29%, and 34% in these years.[23]

A third approach is to follow Brunnermeier et al. (2013) and Bai et al. (2013) who propose and implement a comprehensive measure of funding risk. They estimate that under a severe adverse scenario the 50 largest U.S. bank holding companies stand to lose about 40% of their total funding (including equity). For our paper, a funding shock of this magnitude is arguably on the high side.

In light of these considerations, we take an intermediate value and set λ so that the funding shocks in (7) are equal to 29% of deposits (that is $\lambda/(1+\lambda) = 0.29$). We also set the intensity η to 0.10 and the fire sale loss ϕ to 0.15. For our analysis, these two parameters are only important for guaranteeing that banks fully insure against funding shocks (see (21)).

Next, we set the government bond liquidity services ratio, $1/m$. To identify it, we use (13) and the corresponding series shown in Figure 1, and regress the fed funds-T-bill spread

[22] Campbell (1999) estimates an EIS less than one based on a regression of aggregate consumption growth on the real interest rate. Running this regression within our model would produce an estimate that is even lower—in fact zero—as consumption growth is i.i.d. Our model provides an example where this regression is misspecified due to limited risk sharing.

[23] The numbers are from the U.S. Flow of Funds, Table L.110. Liquid assets are calculated as the sum of Treasury and agency bonds, agency-backed MBS, and federal funds sold and security repos (asset). Deposits and other short-term debt are calculated as the sum of checkable deposits, small time and savings deposits, large time deposits, federal funds and security repos (liability), open market paper, and net interbank liabilities.

on the fed funds rate both in levels and changes. The estimated sensitivities are 0.17 and 0.38. We take a number in the middle of this range and set $1/m = 0.25$.

We compare equilibria across two nominal rate policies. In the first the nominal rate is identically zero. This makes holding liquidity costless, so the model is equivalent to a frictionless one with no funding risk and hence serves as a useful benchmark. In the second policy the nominal rate is 5%, making liquidity costly and constraining leverage. While the model allows for much more complex policy rules, we start with these simple ones in order to convey the main intuition. We consider dynamic policies in Section V.E below.

B. *Risk taking and risk premia*

We begin by looking at risk taking and risk premia as they are directly impacted by monetary policy through its effect on the cost of leverage. The top two panels of Figure 2 show the risky asset portfolio weights of banks (A agents) and depositors (B agents) under the low-rate policy $n_l = 0\%$ (solid red lines) and the high-rate policy $n_h = 5\%$ (dashed black lines). The horizontal axis covers the range of banks' wealth share ω, the model's sole state variable.

Under the high-rate policy n_h, bank leverage is lower throughout the state-space, and especially when banks' wealth share is small. When ω is close to zero, banks' risky asset holdings fall from around ten times net worth under n_l to less than two under n_h. At the other end, when ω is close to one, banks take almost no leverage under either policy as they effectively dominate the economy. At moderate levels of ω between 0.2 and 0.4, where the economy spends most of its time, banks' holdings of risky assets decrease from between two and four times net worth under the low-rate policy n_l to barely above one under the high-rate policy n_h. This represents a substantial decrease in leverage.

As higher nominal rates cause banks to contract their risky asset holdings, depositors must expand theirs. For instance, as the top right panel of Figure 2 shows, when ω is in the 0.2 to 0.4 range, depositors hold between 20% and 40% of their wealth in the risky asset under the low-rate policy n_l, versus almost 100% under the high-rate policy n_h. This shift in the allocation of risk in the direction of the more risk averse depositors amounts to an increase in the risk aversion of the representative investor.

The relationship between portfolio weights and the wealth share ω in Figure 2 is the result of market clearing. When ω is close to either zero or one, a single type of agent dominates the economy, which reduces the opportunity for risk sharing. Agents of the dominant type must hold all their wealth in the endowment claim, whereas agents of the vanishing type can be satisfied with only a small amount of borrowing or lending. Thus, when ω is near zero depositors set prices, so banks take high leverage as long as the nominal rate is not prohibitively high. Conversely when ω is near one banks set prices, so depositors pull back from the risky asset unless a high nominal rate keeps the risk premium sufficiently high.

We see that under the high-rate policy $n_h = 5\%$, holding liquidity to insure against a loss of funding is very costly so that banks take little leverage. This is why under n_h portfolio demand is relatively flat in ω. At even higher levels of the nominal rate, the economy enters financial autarky (see Corollary 2): the credit market shuts down and all agents hold their entire wealth in the risky asset regardless of ω

The bottom two panels of Figure 2 show how the reallocation of risk induced by changing the nominal interest rate influences the Sharpe ratio (bottom left) and risk premium (bottom right) of the risky endowment claim. As noted above, by making leverage more costly higher nominal rates increase effective risk aversion in the economy. This has a strong effect on the Sharpe ratio or price of risk. At moderate levels of ω between 0.2 and 0.4, the price of risk increases from between 0.07 and 0.12 under the low-rate policy n_l to about 0.28 under n_h. This corresponds to an increase in effective risk aversion from between three and five to fourteen.[24]

The bottom right panel of Figure 2 shows that the risk premium largely tracks the Sharpe ratio. In the range of ω between 0.2 and 0.4, it rises from 0.15%–0.30% under the low-rate policy to about 0.57% under the high-rate policy. Note that this is the premium on a claim to the aggregate endowment, which has a relatively low volatility of 2%. By comparison, equity volatility is around 15%. Hence, the equity premium implied by the equilibrium Sharpe ratios is seven to eight times larger than that of the endowment claim, which puts it in the range of standard estimates.

Overall, Figure 2 shows that monetary policy has substantial direct effects on risk taking and risk premia. By raising the cost of holding liquid securities, a higher nominal rate makes taking leverage expensive and causes banks to reduce risk taking. The resulting rise in effective risk aversion increases risk premia and the price of risk by large amounts.

C. Asset prices and volatility

By changing risk premia and the allocation of risk, monetary policy also affects real rates, asset prices, and volatility. These effects are shown in Figure 3. The top left panel focuses on the real deposit rate r. It shows that under the high nominal rate policy $n_h = 5\%$ the real rate is lower than under $n_l = 0\%$ with the difference largest near $\omega = 1$. While it may seem surprising that an increase in the nominal rate decreases the real rate, this is a direct consequence of the increase in effective risk aversion. This increase implies both a higher risk premium and a greater precautionary savings motive. The greater precautionary savings

[24] This effective risk aversion is the one which would produce the same price of risk in a homogeneous economy, i.e. the Sharpe ratio divided by the volatility. Note that even at $\omega = 1$ the Sharpe ratio and the risk premium do not converge to their values in an economy populated only by A agents (banks). The reason is that the risk averse B agents (depositors) remain marginal because they are unconstrained. See Kogan, Makarov, and Uppal (2007) for a discussion.

motive accounts for the decrease in the real rate.[25]

Overall, a higher nominal rate unambiguously results in higher discount rates and lower asset prices. This is shown in the top left panel of Figure 3, which plots the equilibrium price dividend ratio $P/Y = 1/F(\omega)$ of the risky endowment claim. At all levels of ω, the price of the endowment claim is lower under the high-rate policy n_h.

We note that the impact of the nominal rate on valuations depends on the EIS. When the EIS exceeds one, the substitution effect dominates the income effect so that greater risk aversion reduces asset demand and valuations fall. In this case the rise in the risk premium exceeds the fall in the real rate. In contrast, if the EIS is less than one, greater risk aversion counterintuitively causes the valuations of risky assets to increase.

The effect of nominal rates on valuations is strongest near the middle of the state-space where asset prices are about 12% higher when the nominal rate is low. In this region, the lower leverage induced by a high nominal rate has a large impact on the allocation of risk: it causes the overall demand for leverage and supply of deposits to decrease the most. In contrast, when ω is near zero, even a large reduction in leverage per dollar of bank net worth has little effect since bank net worth itself is very low. Similarly, when ω is close to one, the supply of deposits is low regardless of the nominal rate. Even so, valuations continue to be significantly higher at these two extremes under the low-rate policy. The reason is a dynamic effect: agents anticipate lower discount rates in the future when aggregate risk sharing will be higher, and capitalize them into today's asset price.

Figure 3 also plots the volatility of returns σ (bottom left panel). Even though fundamental (cash flow) volatility is constant, return volatility is time varying. Moreover, it exceeds fundamental volatility in a hump-shaped pattern. Under the low-rate policy n_l, return volatility peaks near $\omega = 0.2$ at about 2.7%, which is 35% higher than fundamental volatility.

This excess volatility is the result of changing discount rates, which are determined by a wealth-weighted average of the risky-asset demands of the two types of agents. At low nominal rates and moderate values of ω, banks take significant leverage yet command enough wealth to affect prices. As a result, endowment shocks have a large effect on banks' wealth share ω. Negative cash flow news hit banks disproportionately, causing ω to fall. Moreover, when the nominal rate is low, leverage is very sensitive to ω as we saw in Figure 2. The combination of large movements in bank wealth and a high sensitivity of leverage to bank wealth implies that effective risk aversion and discount rates fluctuate strongly with fundamental shocks, which leads to high volatility. In contrast, when one type of agent dominates the economy or when nominal rates are high, bank wealth and leverage are

[25]Indeed, the same result obtains in a homogeneous economy in a comparative static with respect to risk aversion. Specifically, in a homogeneous economy with RRA γ and EIS ψ, we have $\partial(\mu - r)/\partial\gamma = \sigma^2 > 0$ and $\partial r/\partial\gamma = -\left(\sigma^2/2\right)(1 + 1/\psi) < 0$.

insensitive to shocks and hence there is little variation in discount rates and little excess volatility.

Finally, the bottom right panel of Figure 3 plots the stationary distribution of banks' net worth share ω, which we compute by solving the associated forward Kolmogorov equation. This stationary distribution helps to illustrate the nominal rate's dynamic effects. The impact of the nominal rate on the wealth distribution is significant. The central feature is that under the low-rate policy n_l, the density of ω centers around a higher mean but is also much more disperse. The greater dispersion is a result of the greater leverage, which makes banks' wealth more volatile. The higher mean occurs because banks' greater leverage causes their wealth to grow faster on average.

The volatility and wealth distribution plots in Figure 3 show that low nominal rates are associated with significantly greater endogenous risk. As a result of greater leverage, excess volatility is greater and banks' net worth is much more variable under the low-rate policy n_l. This result illustrates the potential influence of monetary policy on financial stability.

D. *Aggregate liquidity and policy implementation*

The results so far show that monetary policy has a large effect on asset prices. To understand how this effect is achieved, we now analyze the equilibrium supply of liquidity and how the central bank implements its nominal rate policy.

The top two panels of Figure 4 plot aggregate liquidity, scaled by total wealth on the left and bank assets on the right. In both panels, aggregate liquidity is small under the high-rate policy n_h. Since higher nominal rates raise liquidity premia and make holding liquid assets more costly, banks reduce their demand for liquid assets and take less leverage. In contrast, under n_l, where nominal rates are zero, the liquidity premium vanishes and holding liquid assets becomes costless. Banks satiate their demand for liquidity and increase leverage to its unconstrained level.

Looking across the state space, aggregate liquidity scaled by total wealth peaks at moderate values of ω, where aggregate risk sharing is greatest. Scaled by bank assets, aggregate liquidity is highest for low ω, where bank leverage is greatest. The variation in aggregate liquidity is much larger under the zero-rate policy n_l because leverage is much more volatile.

We now look at how the central bank conducts open market operations to achieve its nominal rate target. To do so, we must first specify the evolution of the government bond supply. For simplicity, we assume that the nominal supply of government bonds G_t grows with nominal wealth:

$$\frac{dG_t}{G_t} = \iota_t dt + \frac{dP_t}{P_t}, \tag{29}$$

where $\iota_t = n_t - r_t$ (equation (10)) is the inflation rate. In practice the supply of liquid assets such as retail bank deposits (which we subsume in G_t) actually tends to shrink in response to a nominal rate increase, as noted in Section IV.B. This contraction reinforces the central bank's actions, decreasing the liquidity supply and hence increasing liquidity premia in response to a nominal rate increase. We abstract from this and fix the simple specification in (29) in order to illustrate the mechanics of open market operations.

Substituting (29) into (28), open market operations must follow

$$\frac{dM_t}{M_t} = (n_t - r_t)\,dt + \frac{dP_t}{P_t} + \frac{1}{\alpha_t}\left[\frac{d\Pi_t}{\Pi_t} + \left(\frac{dP_t}{P_t}\right)\left(\frac{d\Pi_t}{\Pi_t}\right)\right] \equiv \mu_{M,t}dt + \sigma_{M,t}dB_t. \quad (30)$$

That is, reserves must grow with nominal wealth while absorbing fluctuations in the demand for liquidity (the term in brackets). To obtain the dynamics of open market operations, we must also take a stance on α_t, the contribution of reserves to total liquidity held by banks (Proposition 3). We look at a single point in time, setting $\alpha = 0.14$ based on pre-crisis data from the U.S. Flow of Funds.[26]

The bottom two panels of Figure 4 show the drift μ_M and shock sensitivity σ_M of open market operations under the two nominal rate policies $n_l = 0\%$ and $n_h = 5\%$. Note from Figure 3 that the economy spends the vast majority of its time near moderate levels of ω where both quantities are relatively flat. At the extremes of ω one type of agent starts to disappear and hence liquidity demand is small and sensitive to ω. Reserves are then very small and their growth rate μ_M is large.

The growth rate of reserves μ_M is higher under the high-rate policy. To achieve a high nominal rate, the central bank must make liquid assets costly to hold. It does so by issuing reserves at a high rate, which causes them to depreciate. In contrast, under the low-rate policy μ_M is low and sometimes turns negative. In this case the central bank makes liquidity less costly by repurchasing reserves over time, causing them to appreciate. When the nominal rate is zero, liquidity is costless and the central bank is following the Friedman rule (1969).

The shock sensitivity of open market operations σ_M is near zero under the high-rate policy n_h because aggregate liquidity is almost constant, as seen in the top panels of Figure 4. Under this policy, banks take little leverage and hence their net worth and demand for liquidity are stable.

In contrast, under the low-rate policy n_l the sensitivity σ_M is positive at low levels of ω and negative at high levels of ω. The explanation lies in the shape of aggregate liquidity

[26] We calculate this number as follows: Looking at commercial banks in Table L.110, from 2004 to 2007 reserves (vault cash plus depository institution reserves) average \$69 billion, while other liquid assets average \$1,621 billion (see the discussion in Section V.A). Applying the liquidity services parameter from Table I implies $\alpha = 0.14$. From 2008 to 2013, this number rises to 0.62 as banks have accumulated large excess reserves. We work with the pre-crisis number.

in the top left panel of Figure 4. When ω is low, a positive endowment shock increases aggregate leverage. To keep the nominal rate stable, the central bank must accommodate the additional demand for liquidity. When ω is higher, positive shocks reduce aggregate leverage, requiring the central bank to "mop up" the excess liquidity.

E. Dynamic policy applications

We now analyze two applications in which the dynamic aspect of monetary policy plays a key role. The first shows how the central bank can stimulate asset prices through forward guidance even when the nominal rate is already at zero. In the second application we implement a "Greenspan put" policy in which the central bank stabilizes asset prices by cutting rates in response to a sequence of bad shocks.

E.1. Forward guidance and the zero lower bound

A zero lower bound arises endogenously in the model. With the nominal rate at zero, the liquidity premium is also zero and banks' demand for leverage becomes satiated. Attempting to push the nominal rate below zero would create an arbitrage opportunity in which banks can raise deposits to invest in risk-free liquid assets. Nevertheless, the central bank can still influence asset prices by changing the course of future nominal rates, i.e. through forward guidance.

Figure 5 illustrates how forward guidance works. The left panel plots two nominal rate policies, a benchmark policy n_0 and a forward guidance policy n_{fg}. Consider a situation in which bank capital has fallen to a low level as in a financial crisis, and as a result the central bank has lowered the nominal rate to zero. Under the benchmark policy n_0 (dashed black line), investors anticipate that the central bank will increase the nominal rate as soon as bank capital has recovered to a value of $\omega = 0.25$. Under the forward guidance policy n_{fg} (solid red line), the central bank commits to delaying the rate increase until $\omega = 0.3$. Hence, under forward guidance, rates are expected to remain low for a longer period.

The right panel of Figure 5 plots the ratio of the prices of the risky asset under the two policies, P_{fg}/P_0. Consider the region $\omega < 0.25$ in which nominal rates have hit the zero lower bound under both policies. The plot shows that the central bank is nevertheless able to induce an increase in asset prices by guiding down expectations of future rates under policy n_{fg}. Forward guidance has a substantial impact on asset prices. For example, for $\omega = 0.25$ the price of the endowment claim is around 4% higher under the forward guidance policy n_{fg} than under the benchmark policy n_0.

Guiding future nominal rates down increases prices by inducing a decrease in future discount rates. Investors expect that assets will be worth more in the future, and they are therefore willing to pay more for them today. This effect is purely dynamic, it does not work by changing the cost of taking leverage today since this cost is already zero.

The impact of forward guidance on asset prices peaks close to the point where nominal rates are set to lift off. In this region, the timing delay under forward guidance pushes up valuations by as much as 6%. This means that a policy reversal would provoke a large correction, similar to the "taper tantrum" of mid-2013 (see Sahay et al., 2014).

Finally, prices remain higher under forward guidance even at values of $\omega > 0.3$, where rates under the two policies are the same. This happens because investors take into account the positive impact of forward guidance on valuations when bank capital is low. This result illustrates the two-sided nature of forward guidance: The same commitment that allows the central bank to cushion downturns causes it to amplify booms.

E.2. The "Greenspan put"

As our second application of a dynamic policy, we implement a "Greenspan put".[27] We interpret a Greenspan put as a policy that reduces nominal interest rates in the event of a large-enough sequence of negative shocks. We capture this policy with the nominal rate rule

$$n_{gp}(\omega_t) = \min\left\{0.05, \frac{0.05}{0.3}\omega_t\right\} \tag{31}$$

and compare it to the constant-rate benchmark $n_0(\omega_t) = 0.04$. The top left panel of Figure 6 plots these two policy rules. Under n_{gp}, the nominal rate rises from 0% at $\omega = 0$ at a constant slope until it reaches 5% at $\omega = 0.3$, and then levels off. This implies that a sequence of negative shocks that pushes banks' net worth share ω below 0.3 triggers progressive rate cuts. We set the level of the constant benchmark n_0 so that the two policies have similar unconditional averages (integrated against the stationary distribution of ω).

The top right panel of Figure 6 compares the risky asset's price-dividend ratio under the two policies. Compared to the constant benchmark, under the Greenspan put policy valuations are higher when ω is low, and lower when ω is high. This pattern mirrors that of the nominal rate. When bank capital is high, the nominal rate is higher under the Greenspan put policy, so valuations are lower. As ω approaches the "strike" of the put near 0.3 and rate cuts become imminent, valuations under the two policies converge. As ω falls below 0.3, valuations under n_{gp} flatten out and even mildly increase, whereas under n_0 they fall steadily.

[27]The term dates to the late 1990s when critics faulted Federal Reserve chairman Alan Greenspan for "encouraging excessive risk taking by creating what came to be called 'the Greenspan put', that is, the belief that the Fed would, if necessary, support the economy and therefore the stock market" (Blinder and Reis, 2005).

This illustrates how the central bank supports asset prices under the Greenspan put policy by cutting rates and inducing greater bank leverage. However, should ω continue to fall, there is little room for increasing leverage further. High valuations can no longer be supported, and prices start to fall steeply. By the time ω nears zero, prices are about the same under the two policies. Thus, the Greenspan put policy stabilizes prices in a moderate downturn but cannot forestall a severe price decline in a highly adverse scenario.

The bottom left panel of Figure 6 plots the corresponding risk premia. When bank capital is high, the higher nominal rates of the Greenspan put policy result in a higher risk premium. As ω declines towards 0.3, the stabilization effect of the policy results in lower risk premia. Once ω falls below 0.3, the risk premium drops precipitously as a result of the aggressive rate cutting. However, when ω nears zero, prices are set to fall even more steeply than under the benchmark policy, so the risk premium under the Greenspan put eventually exceeds that under the benchmark policy.

The bottom right panel of Figure 6 plots the corresponding return volatility. The pattern here is striking. Under the Greenspan put policy n_{gp}, volatility is lower when ω is high. This is due to the higher nominal rate in this region, which reduces risk taking and stabilizes effective risk aversion. As ω declines towards 0.3, volatility dips further as the prospect of intervention keeps prices from falling. Past 0.3, the put goes "into the money" and the rate cutting kicks in, causing volatility to fall even more so that it briefly dips below fundamental volatility. The Greenspan put is thus able to reduce volatility in moderate downturns. However, if ω declines even further, the temporary support runs out, and volatility spikes sharply due to the high level of leverage built up under the Greenspan put policy. Thus, although the model predicts that low nominal rates increase volatility in the long run, the short-run relationship depends on how the policy rule responds to economic shocks.

The results in Figure 6 convey the basic tradeoff that underlies the Greenspan put. On one hand, the policy achieves short-run stability by boosting leverage in moderate downturns. However, that same leverage build-up leads to instability if the downturn proves severe. Moreover, greater leverage raises the likelihood that bank capital will fall to the low levels associated with a severe downturn. These results formalize the concern that the Greenspan put has short-term benefits but long-term costs (see Blinder and Reis, 2005).

VI. Nominal rate shocks and the real economy

In our baseline model the nominal rate follows a known policy rule that is a function of the state variable ω_t. As there are no independent nominal rate shocks, we have so far analyzed the impact of monetary policy by comparing across equilibria with different policy rules. In this section we introduce shocks to the nominal rate that are independent of ω_t, which allows us to study propagation within a single equilibrium.

When a nominal rate shock occurs, the central bank changes the nominal rate independently of the endowment shock. Let $n_0(\omega_t)$ be the benchmark policy rule, which can be interpreted as the nominal rate that would prevail absent independent rate shocks. To introduce these shocks, let the actual nominal rate n_t follow the process

$$dn_t = -\kappa_n\Big[n_t - n_0(\omega_t)\Big]dt + \sigma_n\sqrt{(n_t - \underline{n})(\overline{n} - n_t)}dB_t^n, \tag{32}$$

where dB_t^n is independent of the other shocks in the economy. Under this process, the nominal rate reverts towards the benchmark $n_0(\omega_t)$ at the rate κ_n. The diffusive loading ensures that n_t is bounded below by $\underline{n}$ and above by $\overline{n}$, which keeps it from straying too far from the benchmark rule. Given this structure, the nominal rate n_t becomes an additional state variable in the model.

In addition to asset prices and risk premia, we are also interested in the effects of monetary policy on economic activity. To examine these effects, we further extend the model by adopting a production framework. This allows us to look at the propagation of nominal rate changes to investment and output.

In introducing production, we follow Brunnermeier and Sannikov (2014a) and replace the endowment process (1) with the capital accumulation equation

$$\frac{dk_t}{k_t} = [\phi(i_t) - \delta]\,dt + \sigma_k dB_t^k, \tag{33}$$

where ϕ is a concave function that captures investment adjustment costs, i_t is the investment rate, and δ is the depreciation rate. Output from capital is produced at a rate

$$Y_t = ak_t, \tag{34}$$

where a is productivity. As in the baseline model, the economy is homogeneous in Y_t, or equivalently k_t. Agents of both types trade capital, which represents the economy's risky asset. Its total (real) value is $P_t = q_t k_t$, where $q_t = q(\omega_t, n_t)$ is the price of a unit of capital, with endogenous dynamics

$$\frac{dq_t}{q_t} = \mu_{q,t}dt + \sigma'_{q,t}\begin{bmatrix} dB_t^k \\ dB_t^n \end{bmatrix} \equiv \mu_{q,t}dt + \sigma'_{q,t}dB_t. \tag{35}$$

The return to holding capital is given by

$$dR_t = \left(\frac{a-i_t}{q_t} + \phi\left(i_t\right) - \delta + \mu_{q,t} + \sigma'_{q,t}\begin{bmatrix}\sigma_k \\ 0\end{bmatrix}\right)dt + \left(\sigma_{q,t} + \begin{bmatrix}\sigma_k \\ 0\end{bmatrix}\right)'\begin{bmatrix}dB_t^k \\ dB_t^n\end{bmatrix} \tag{36}$$

$$\equiv \mu_t dt + \sigma'_t dB_t. \tag{37}$$

The expected return on capital μ_t is given by cash flows generated per dollar of capital net of investment expenditures $\left(a-i_t\right)/q_t$ (in the baseline model this is the dividend yield F_t), the accumulation of new capital $\phi\left(i_t\right)$, the depreciation of existing capital $-\delta$, the expected price appreciation of capital $\mu_{q,t}$, and the covariance between the price of capital and cash flows. Return volatility σ_t is the sum of price volatility $\sigma_{q,t}$ and cash flow volatility σ_k.

The optimization problem of each agent now includes investment choice. This choice boils down to maximizing the expected return in (36), which gives

$$q_t\phi'\left(i_t\right) = 1. \tag{38}$$

This is Tobin's (1969) q-theory of investment. It states that investment and economic growth depend on the price of capital. Because monetary policy impacts asset prices, it affects investment and economic growth.

As discussed in Section V, in addition to raising risk premia higher nominal rates in our baseline model also lower real rates due to precautionary savings. In conventional models nominal and real rates move together because output is not fixed in the short run: it falls temporarily following a nominal rate increase and then recovers, causing the real rate to rise.

To aid the fit of the model, we follow the literature and introduce such a transitory component of output, an output gap, which we denote by $o\left(\omega_t, n_t\right)k_t$. We do so in a narrowly targeted way. We construct the output gap so that the real rate is simply unaffected (rather than increased) by nominal rate shocks, i.e. we require that $\partial r_t\left(\omega_t, n_t\right)/\partial n_t = 0$ in equilibrium, and set the output gap to zero at the benchmark nominal rate, i.e. $o\left(\omega_t, n_0\left(\omega_t\right)\right) = 0$. This implies that the output gap turns negative when the nominal rate is shocked up, positive when it is shocked down, and that it subsequently recovers.[28]

Total output is then output from capital (34) plus the output gap, and hence the goods-market clearing condition (17) becomes

$$\omega_t c_t^A + \left(1-\omega_t\right)c_t^B = \frac{a-i_t}{q_t} + \frac{o_t}{q_t}. \tag{39}$$

As with seigniorage income, we assume for simplicity that the output gap is borne by house-

[28] A natural interpretation is that the output gap comes from the labor market, capturing the deviation of labor income from its full-employment level.

holds in proportion to their wealth. We emphasize that as in the baseline model, monetary policy affects asset prices entirely through risk premia as the output gap has no impact on the cash flows of the risky asset (capital).

The rest of the model is unchanged. Appendix B presents the solution of the extended model, which largely follows that of the baseline model.

A. *Extended model parameter values*

To facilitate comparison with the baseline model results, we keep all common parameters at their levels in Table I. The values of the new parameters are in Table II. As before, we set the volatility of cash flows to 2% with $\sigma_k = 0.02$. We set productivity $a = 0.03$, depreciation $\delta = 0.0075$. We specify the adjustment cost function as $\phi(i_t) = 1/\varphi\sqrt{1 + 2\varphi i} - 1$ and use $\varphi = 1$, which implies modest quadratic adjustment costs.

Turning to the parameters of the nominal rate process (32), we set $\underline{n} = 0$ and $\overline{n} = 0.05$, which bounds the nominal rate between the levels we considered for the results of the baseline model. For simplicity, we set a constant benchmark rule $n_0 = 0.01$. We set $\kappa_n = 0.2$, which implies that the annual persistence of the nominal rate is 0.8.[29] Finally, we set the volatility parameter $\sigma_n = 0.5$, which implies that the annualized standard deviation of nominal rate changes at the benchmark is 100 basis points (bps).

B. *Nominal rate shocks and financial markets*

We begin the analysis by looking at the effects of nominal rate shocks on risk taking and risk premia. We do so by tracing out the impulse responses of demand for the risky asset, risk premia, and Sharpe ratios to a one-time 100 bps shock to n_t from its benchmark value when banks' net worth ω_t is at its stochastic steady state.

Figure 7 plots these impulse responses. The top left plot shows the path of the nominal rate, which rises initially from 100 bps to 200 bps and then declines over time. The plot to the right shows the response of banks' net worth share ω, which falls by 22 bps. It does so because the increase in the nominal rate causes the value of the risky asset to fall. As banks are levered, their wealth falls faster than that of depositors, and hence their share of total wealth declines.

Since risk premia in the model depend strongly on banks' share of wealth, this drop produces a second round of effects: lower bank net worth reduces demand for leverage, causing risk premia to rise further and asset prices to fall further. The literature calls this

[29] The annual AC(1) coefficient of the fed funds rate from 1955 to 2010 (1987 to 2010) is 0.79 (0.78). This number is also very similar to the persistense of changes in investor expectations about nominal rates based on data from the Blue Chip survey (Nakamura and Steinsson, 2013).

amplification mechanism a balance sheet channel or financial accelerator (Bernanke, Gertler, and Gilchrist, 1999). Thus our extended model features both the direct effect present in the baseline model, and its amplification via bank balance sheets.

The impulse response of banks' net worth in Figure 7 highlights a novel feature of the interaction between changes in the nominal rate and the bank balance sheet channel. At a high nominal rate, taking leverage becomes more expensive and hence banks' net worth grows more slowly. As a result, the gap between banks' net worth and its steady state value is much more persistent than the nominal rate itself. This dynamic effect further adds to the strength of the balance sheet channel: The increased nominal rate not only reduces banks' net worth today (as would any wealth shock), but also far into the future.

The middle panels of Figure 7 show the impact of the nominal rate on the risky asset demand of banks (left panel) and depositors (right panel). As in the baseline model, the higher nominal rate induces a decrease in banks' leverage, which falls by 0.21, and a corresponding increase in depositors' risky asset portfolio weight, which rises by 0.11. These numbers correspond a contraction of about 8.5% in bank risk taking.

The bottom panels of Figure 7 show that as in the baseline model, this reduction in aggregate risk bearing capacity results in a higher risk premium and price of risk. The 100 bps nominal rate shock causes the risk premium to rise from 27 bps to 36 bps. The Sharpe ratio rises from 0.13 to 0.17, which corresponds to effective risk aversion rising roughly from 6.25 to 8.50. As we show next, the rise in risk premia leads to a fall in asset prices, investment, and economic growth.

C. Nominal rate shocks and economic activity

Figure 8 shows the impulse responses of various macroeconomic variables to the nominal rate shock. The top right panel plots the response of the price of capital q. As the nominal rate increases by 100 bps, the rise in risk premia seen in Figure 7 causes the value of the aggregate capital stock to drop by 42 bps. This result is consistent with the finding in Bernanke and Kuttner (2005) that a positive nominal rate shock is associated with a substantial negative stock market return contemporaneously and positive future excess returns several years out. Indeed, the magnitudes are similar after adjusting for the stock market's seven to eight times higher volatility relative to the aggregate capital stock we price. The drop reverts slowly over time, although q remains persistently below its initial value due to the slow growth of banks' net worth share highlighted in Figure 7. The substantial response of the capital price to the nominal rate shock reflects the sizable increase in effective risk aversion that we discussed above.

The drop in the capital price results in a drop in investment as dictated by q-theory (see (38)). As the middle-left panel of Figure 8 shows, the investment rate falls by 42 bps, dipping below the depreciation rate δ, so that for one period the economy in effect disinvests. Subsequently, investment starts to rebound but remains persistently below its initial value.

The reduction in the investment rate decreases the rate of capital accumulation. This results in a lower rate of output growth as seen in the middle-right panel of Figure 8. The plot shows $\log Y$, output produced by capital, which does not include the output gap. The dashed line shows the cumulative growth of $\log Y$ in the steady state, while the solid red line shows its response to the 100 bps nominal rate shock. The plot shows that output from capital grows more slowly following the shock. The gap relative to the steady state continues to widen over time and grows into a roughly 200 bps permanent loss of output.

The bottom two panels of Figure 8 show the output gap o and the cumulative growth of consumption, $\log C$. The output gap is negative following the nominal rate shock as expected and reverts back as the shock itself subsides. This reduction comes in addition to the fall in output from capital Y. However, while the loss in output from capital grows over time, the output gap is transitory. Despite the loss of output, consumption actually increases initially. This occurs because of the drop in investment induced by the increased risk premium. Indeed, the initial drop in investment is larger than the initial drop in output, implying increased consumption.[30] Consumption then grows more slowly as investment begins to normalize while capital accumulation remains low. In the long run, consumption is lower than in the steady state, reflecting the permanently lower level of output.

Overall, Figures 7 and 8 show that by increasing the cost of holding liquidity, a positive shock to the nominal rate causes risk premia to rise and asset prices to fall, which in turn leads to a downturn in investment and economic growth.

D. *The term structure of nominal rates*

To better understand our model, it is interesting to look at the term structure of nominal interest rates. This analysis is informative about interest rates and risk premia at various horizons and their response to nominal rate shocks.

Let $p_t^\tau = p^\tau(\omega_t, n_t)$ be the nominal price and $y_t^\tau = -(1/\tau)\log p_t^\tau$ be the yield at time t of a bond that pays one dollar at the maturity date $t+\tau$ (equivalently, the bond pays $\pi_{t+\tau}$ units of consumption). For simplicity, we assume the bond does not provide any special liquidity services and therefore price its cash flow using the discount factor implied by the fed funds rate. Doing so avoids having to make assumptions about how bonds' liquidity services vary with maturity. The bond's yield is therefore equivalent to the fixed rate on an equal-maturity interest rate swap whose floating rate is the fed funds rate.

[30] The initial rise in consumption is arguably stark. Using adjustment costs over the change in the investment rate rather than its level, as used in the literature (e.g. Christiano et al., 2014), would smooth the adjustment in investment and hence also in consumption.

As markets in the model are incomplete due to imperfect risk sharing, p_t^τ is not pinned down by the prices of the existing securities. We therefore price the nominal bond by calculating the amount that depositors, who are unconstrained, are willing to pay for it at the margin. Note that the nominal bond price is a function of three state variables. To deal with this added level of complexity, we construct a routine for pricing a sequence of nominal bonds recursively, starting from the terminal condition and working backwards. The details are in Appendix B.

Figure 9 depicts the model's nominal term structure in steady state and in response to a 100 bps nominal rate shock. We continue to use the same parameters and initial conditions as in Figures 7 and 8. The top left panel of Figure 9 shows the expected path of the nominal rate over time and is the same as the impulse response showin in Figures 7 and 8.

The top right panel plots the yields y^τ against their maturity τ, i.e. the yield curve. The dashed line plots the yield curve at the steady state. The steady state yield curve has a pronounced upward slope. The short end is pinned down at the benchmark nominal rate of 100 bps, whereas at the long end yields exceed 150 bps. As short rate expectations are flat in steady state, the difference in yields is due solely to a term premium. The plot therefore shows that our model generates a substantial term premium.

Our model generates this term premium because higher nominal rates imply a higher cost of liquidity and therefore decreased risk sharing and higher risk premia. Thus these are high marginal utility states, and because nominal bond price fall in these states, agents view nominal bonds as risky. Agents therefore demand a risk premium to hold long-term nominal bonds and this generates a substantial term premium. Thus the presence of a liquidity premium allows our model to capture the upward sloping shape of the yield curve.

The bottom left plot of Figure 9 shows the forward rates implied by the nominal yield curve. The dashed line shows the steady state forward curve. As the yield curve is upward-sloping, the forward rates slope upward even more steeply, exceeding 170 bps at the long end. The difference between these forward rates and the expected future short rate gives the term structure of forward premia, the risk premia at each horizon, which is plotted in the lower right panel of Figure 9. Because the expected short rate is constant in steady state, this is just the forward curve shifted down by the benchmark short rate. It confirms that term premia in the model are substantial and increasing with horizon.

The solid red lines in each panel of Figure 9 show responses to the nominal rate shock. On the top right, the yield curve becomes much flatter, sloping up slightly at the short end and down at the long end. The forward curve displays the same pattern. Both curves are flat due to the combination of a decreasing expected short rate and an increasing term premium.

The forward premium curve in the lower right panel isolates the term premium component by subtracting the expected short rate from the forward rates. Following the nominal rate shock, forward premia become even more steeply increasing with horizon. The rise in forward premia is greatest at medium horizons and roughly 10 bps at the long end. This substantial increase in long-horizon forward premia in response to a nominal rate shock is consistent with the findings of Gertler and Karadi (2014) and Hanson and Stein (2014).

VII. Conclusion

There is a growing consensus that monetary policy has a powerful impact on the prices of risky assets and the stability of financial markets. Indeed, many central bank interventions are aimed not just at the level of the interest rate but also at the risk premium component of the cost of capital.

We develop a dynamic asset pricing framework that captures the effect of monetary policy on risk premia. In our framework, the central bank sets the nominal interest rate, which drives the liquidity premium in financial markets. This affects the cost of leverage for financial institutions who hold liquid securities to protect against a loss of funding. Low nominal rates lead to greater leverage, lower risk premia and higher asset prices, and more volatility. Our analysis further shows that by influencing risk premia, monetary policy can have a large impact on real investment and economic growth.

We examine a number of policy interventions that are conducted through the nominal interest rate, including high versus low interest rate rules, forward guidance, a "Greenspan put", and unexpected rate changes. Our framework can also be used to analyze unconventional policy interventions that are conducted through means other than the nominal interest rate. For example, quantitative easing reduces effective risk aversion by deploying the central bank's balance sheet to increase overall risk taking. And in its capacity as lender of last resort, the central bank provides liquidity directly, which reduces banks' need for liquidity buffers and enables them to expand credit supply. We leave these applications for future work.

References

Acharya, Viral V, Douglas Gale, and Tanju Yorulmazer, 2011, Rollover risk and market freezes, *The Journal of Finance* 66, 1177–1209.

Acharya, Viral V., Philipp Schnabl, and Gustavo Suarez, 2013, Securitization without risk transfer, *Journal of Financial Economics* 107, 515 – 536.

Adrian, Tobias, and Nina Boyarchenko, 2012, Intermediary leverage cycles and financial stability, *Working paper* .

Adrian, Tobias, and Hyun Song Shin, 2010, Financial intermediaries and monetary economics, in Benjamin M. Friedman, and Michael Woodford, eds., *Handbook of Monetary Economics*, volume 3, first edition, chapter 12, 601–650 (Elsevier).

Afonso, Gara, Anna Kovner, and Antoinette Schoar, 2011, Stressed, not frozen: The federal funds market in the financial crisis, *The Journal of Finance* 66, 1109–1139.

Allen, Franklin, and Douglas Gale, 1994, Limited market participation and volatility of asset prices, *The American Economic Review* 933–955.

Ashcraft, Adam, Nicolae Garleanu, and Lasse Heje Pedersen, 2011, Two monetary tools: Interest rates and haircuts, in *NBER Macroeconomics Annual 2010, Volume 25*, 143–180 (University of Chicago Press).

Bai, Jennie, Arvind Krishnamurthy, and Charles-Henri Weymuller, 2013, Measuring the liquidity mismatch in the banking sector, *Working paper* .

Basel Committee on Banking Supervision, 2013, Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools, *Bank for International Settlements* .

Basel Committee on Banking Supervision, 2014, Basel III: The Net Stable Funding Ratio, *Bank for International Settlements* .

Bech, Morten L., and Elizabeth Klee, 2011, The mechanics of a graceful exit: Interest on reserves and segmentation in the federal funds market, *Journal of Monetary Economics* 58, 415 – 431.

Bekaert, Geert, Marie Hoerova, and Marco Lo Duca, 2013, Risk, uncertainty and monetary policy, *Journal of Monetary Economics* 60, 771–788.

Bernanke, Ben S., 1983, Nonmonetary effects of the financial crisis in the propagation of the great depression, *The American Economic Review* 73, 257–276.

Bernanke, Ben S., 2013, Long-term interest rates At the Annual Monetary/Macroeconomics Conference: The Past and Future of Monetary Policy, sponsored by Federal Reserve Bank of San Francisco.

Bernanke, Ben S., and Alan S Blinder, 1988, Credit, money, and aggregate demand, *The American Economic Review* 78, 435–439.

Bernanke, Ben S., and Alan S Blinder, 1992, The federal funds rate and the channels of monetary transmission, *The American Economic Review* 82, 901–921.

Bernanke, Ben S., and Mark Gertler, 1989, Agency costs, net worth, and business fluctuations, *The American Economic Review* 14–31.

Bernanke, Ben S., and Mark Gertler, 1995, Inside the black box: The credit channel of monetary policy transmission, *Journal of Economic Perspectives* 9, 27–48.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, 1999, The financial accelerator in a quantitative business cycle framework, *Handbook of macroeconomics* 1, 1341–1393.

Bernanke, Ben S., and Kenneth N. Kuttner, 2005, What explains the stock market's reaction to federal reserve policy?, *The Journal of Finance* 60, 1221–1257.

Bhattacharya, Sudipto, and Douglas Gale, 1987, Preference shocks, liquidity, and central bank policy, in W.A. Barnett, and K.J. Singleton, eds., *New Approaches to Monetary Economics*, 69–88 (Cambridge University Press).

Blinder, Alan S, and Ricardo Reis, 2005, Understanding the Greenspan standard, *The Greenspan Era: Lessons for the Future* 25–27.

Board of Governors of the Federal Reserve System, 2014, Federal banking regulators finalize liquidity coverage ratio, *Federal Register* 79, 61439–61541.

Brunnermeier, Markus, Arvind Krishnamurthy, and Gary Gorton, 2013, Liquidity mismatch measurement, in *Risk Topography: Systemic Risk and Macro Modeling* (University of Chicago Press).

Brunnermeier, Markus K, and Lasse Heje Pedersen, 2009, Market liquidity and funding liquidity, *Review of Financial studies* 22, 2201–2238.

Brunnermeier, Markus K., and Yuliy Sannikov, 2014a, A macroeconomic model with a financial sector, *American Economic Review* 104, 379–421.

Brunnermeier, Markus K., and Yuliy Sannikov, 2014b, The I theory of money, *Working paper* .

Caballero, Ricardo J, and Emmanuel Farhi, 2013, A model of the safe asset mechanism (SAM): Safety traps and economic policy, *NBER working paper* .

Campbell, John Y, 1999, Asset prices, consumption, and the business cycle, *Handbook of macroeconomics* 1, 1231–1303.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno, 2014, Risk shocks, *American Economic Review* 104, 27–65.

Coval, Joshua, and Erik Stafford, 2007, Asset fire sales (and purchases) in equity markets, *Journal of Financial Economics* 86, 479–512.

Cúrdia, Vasco, and Michael Woodford, 2009, Credit frictions and optimal monetary policy, *Working Paper* .

Dell'Ariccia, Giovanni, Luc Laeven, and Gustavo Suarez, 2013, Bank leverage and monetary policy's risk-taking channel: Evidence from the United States, *Working paper* .

Diamond, Douglas W., and Philip H. Dybvig, 1983, Bank runs, deposit insurance, and liquidity, *Journal of Political Economy* 91, pp. 401–419.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, 2014, The deposits channel of monetary policy, *Working paper* .

Duffie, Darrell, and Larry G Epstein, 1992, Stochastic differential utility, *Econometrica: Journal of the Econometric Society* 353–394.

Dumas, B, 1989, Two-person dynamic equilibrium in the capital market, *Review of Financial Studies* 2, 157–188.

Epstein, Larry G, and Stanley E Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica: Journal of the Econometric Society* 937–969.

Farhi, Emmanuel, and Iván Werning, 2013, A theory of macroprudential policies in the presence of nominal rigidities, Technical report, National Bureau of Economic Research.

Financial Stability Oversight Council, 2014, 2014 Annual report, *US Government Printing Office* .

Friedman, Milton, 1969, The optimum quantity of money, in Milton Friedman, ed., *The optimum quantity of money and other essays* (Aldine Publishing Company, Chicago).

Gârleanu, Nicolae, and Stavros Panageas, 2008, Young, old, conservative and bold: The implications of heterogeneity and finite lives for asset pricing, *Working paper* .

Gârleanu, Nicolae, and Lasse Heje Pedersen, 2011, Margin-based asset pricing and deviations from the law of one price, *Review of Financial Studies* 24, 1980–2022.

Geanakoplos, John, 2003, Liquidity, default, and crashes endogenous contracts in general, in *Advances in economics and econometrics: theory and applications: eighth World Congress, Volume II*, 170–205.

Gertler, Mark, and Peter Karadi, 2011, A model of unconventional monetary policy, *Journal of Monetary Economics* 58, 17–34.

Gertler, Mark, and Peter Karadi, 2014, Monetary policy surprises, credit costs, and economic activity, *American Economic Journal: Macroeconomics, forthcoming* .

Gertler, Mark, and Nobuhiro Kiyotaki, 2010, Financial intermediation and credit policy in business cycle analysis, in Benjamin M. Friedman, and Michael Woodford, eds., *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics*, chapter 11, 547–599 (Elsevier).

Gilchrist, Simon, and Egon Zakrajšek, 2012, Credit spreads and business cycle fluctuations, *The American Economic Review* 102, pp. 1692–1720.

Gorton, Gary, and George Pennacchi, 1990, Financial intermediaries and liquidity creation, *The Journal of Finance* 45, 49–71.

Gorton, Gary B, and Guillermo Ordoñez, 2014, Collateral crises, *The American Economic*

Review .

Greenwood, Robin, Sam Hanson, and Jeremy Stein, 2013, A comparative-advantage approach to government debt maturity, *Journal of Finance, forthcoming* .

Gromb, Denis, and Dimitri Vayanos, 2002, Equilibrium and welfare in markets with financially constrained arbitrageurs, *Journal of Financial Economics* 66, 361 – 407.

Hanson, Samuel G, and Jeremy C Stein, 2014, Monetary policy and long-term real rates, *Journal of Financial Economics, forthcoming* .

He, Zhiguo, and Arvind Krishnamurthy, 2011, A model of capital and crises, *The Review of Economic Studies* .

He, Zhiguo, and Arvind Krishnamurthy, 2013, Intermediary asset pricing, *American Economic Review* 103, 732–70.

Holmström, Bengt, and Jean Tirole, 1998, Private and public supply of liquidity, *Journal of Political Economy* 106, 1–40.

Investment Company Institute, 2014, *Investment Company Fact Book* (Investment Company Institute).

Jermann, Urban, and Vincenzo Quadrini, 2012, Macroeconomic effects of financial shocks, *American Economic Review* 102, 238–71.

Jiménez, Gabriel, Steven Ongena, José-Luis Peydró, and Jesús Saurina, 2014, Hazardous times for monetary policy: What do twenty-three million bank loans say about the effects of monetary policy on credit risk-taking?, *Econometrica* 82, 463–505.

Kacperczyk, Marcin, and Philipp Schnabl, 2013, How safe are money market funds?, *The Quarterly Journal of Economics* .

Kashyap, Anil K, Raghuram Rajan, and Jeremy C Stein, 2002, Banks as liquidity providers: An explanation for the coexistence of lending and deposit-taking, *The Journal of Finance* 57, 33–73.

Kashyap, Anil K, and Jeremy C Stein, 1994, Monetary policy and bank lending, in *Monetary policy*, 221–261 (The University of Chicago Press).

Kashyap, Anil K, and Jeremy C. Stein, 2000, What do a million observations on banks say about the transmission of monetary policy?, *The American Economic Review* 90, pp. 407–428.

Kashyap, Anil K, Jeremy C Stein, and David W Wilcox, 1993, Monetary policy and credit conditions: Evidence from the composition of external finance, *American Economic Review* 83.

Keeley, Michael C., 1990, Deposit insurance, risk, and market power in banking, *The American Economic Review* 80, pp. 1183–1200.

Kiyotaki, Nobuhiro, and John Moore, 1997, Credit cycles, *Journal of Political Economy* 105.

Kogan, Leonid, Igor Makarov, and Raman Uppal, 2007, The equity risk premium and the riskfree rate in an economy with borrowing constraints, *Mathematics and Financial Economics* 1, 1–19.

Krishnamurthy, Arvind, Stefan Nagel, and Dmitry Orlov, 2014, Sizing up repo, *The Journal of Finance* 69, 2381–2417.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen, 2012, The aggregate demand for treasury debt, *Journal of Political Economy* 120, 233–267.

Kuo, Dennis, David Skeie, and James Vickery, 2012, A comparison of Libor to other measures of bank borrowing costs, *Working paper* .

Landier, Augustin, David Sraer, and David Thesmar, 2013, Banks exposure to interest rate risk and the transmission of monetary policy, Technical report, National Bureau of Economic Research.

Longstaff, Francis A., and Jiang Wang, 2012, Asset pricing and the credit market, *Review of Financial Studies* 25, 3169–3215.

Lucas, Robert E., 1990, Liquidity and interest rates, *Journal of economic theory* 50, 237–264.

Moreira, Alan, and Alexi Savov, 2014, The macroeconomics of shadow banking, *Working paper* .

Nagel, Stefan, 2014, The liquidity premium of near-money assets, *Working paper* .

Nakamura, Emi, and Jón Steinsson, 2013, High frequency identification of monetary non-neutrality, *NBER working paper* .

Rajan, Raghuram G, 2011, *Fault lines: How hidden fractures still threaten the world economy* (Princeton University Press).

Sahay, Ratna, Vivek Arora, Thanos Arvanitis, Hamid Faruqee, Papa N'Diaye, and Tommaso Mancini Griffoli, 2014, Emerging market volatility: Lessons from the taper tantrum, Technical report, International Monetary Fund.

Scharfstein, David, and Adi Sunderam, 2014, Market power in mortgage lending and the transmission of monetary policy, *Working paper* .

Shleifer, Andrei, and Robert W. Vishny, 1997, The limits of arbitrage, *The Journal of Finance* 52, 35–55.

Stein, Jeremy C., 1998, An adverse-selection model of bank asset and liability management with implications for the transmission of monetary policy, *The RAND Journal of Economics* 29, pp. 466–486.

Stein, Jeremy C, 2012, Monetary policy as financial stability regulation, *The Quarterly Journal of Economics* 127, 57–95.

Stein, Jeremy C, 2013, Liquidity regulation and central banking, At the "Finding the Right Balance" 2013 Credit Markets Symposium sponsored by the Federal Reserve Bank of Richmond, Charlotte, North Carolina.

Stein, Jeremy C, 2014, Incorporating financial stability considerations into a monetary policy framework, At the International Research Forum on Monetary Policy, Washington, D.C.

Tobin, James, 1969, A general equilibrium approach to monetary theory, *Journal of money, credit and banking* 1, 15–29.

Wang, Jiang, 1996, The term structure of interest rates in a pure exchange economy with heterogeneous investors, *Journal of Financial Economics* 41, 75–110.

Woodford, Michael, 1990, Public debt as private liquidity, *The American Economic Review* 80, pp. 382–388.

Woodford, Michael, 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy* (Princeton University Press).

Yellen, Janet L, 2011, Assessing potential financial imbalances in an era of accommodative monetary policy At the 2011 International Conference: Real and Financial Linkage and Monetary Policy, Bank of Japan, Tokyo, Japan.

Appendix

A. Baseline model

Return dynamics. We begin by writing down the dynamics of returns:

$$dR = \frac{dY/F}{Y/F} + Fdt = \frac{dY}{Y} - \frac{dF}{F} - \left(\frac{dY}{Y}\right)\left(\frac{dF}{F}\right) + \left(\frac{dF}{F}\right)^2 + Fdt \tag{A.1}$$

$$\mu = \mu_Y + F - \frac{F_\omega}{F}\omega\left(1-\omega\right)\left(\mu_\omega + \sigma_\omega\sigma_Y\right) + \left[\left(\frac{F_\omega}{F}\right)^2 - \frac{1}{2}\frac{F_{\omega\omega}}{F}\right]\omega^2\left(1-\omega\right)^2\sigma_\omega^2 \tag{A.2}$$

$$\sigma = \sigma_Y - \frac{F_\omega}{F}\omega\left(1-\omega\right)\sigma_\omega, \tag{A.3}$$

where $F = Y/P$ is the dividend yield.

Optimization problem. Let $\overline{m} > 1$ be the number of dollars worth of redemptions that a dollar of reserves can backstop. We will link $\overline{m}$ to the liquidity services multiplier m. The two differ only because reserves have the added advantage over government bonds of requiring fewer dollars of funding per unit of liquidity services, that is $m > \overline{m}$.

Dropping agent and time subscripts, an agent's wealth dynamics are

$$\begin{aligned}\frac{dW}{W} &= \left[r - c + w_S\left(\mu - r\right) + w_G\left(r^g - r\right) + w_M\left(\frac{d\pi}{\pi} - r\right) + \Pi\frac{n}{m}\right]dt + w_S\sigma dB \\ &\quad - \frac{\phi}{1-\phi}\max\left\{-\frac{\lambda}{1+\lambda}w_D - \left(w_G + \overline{m}w_M\right), 0\right\}dN.\end{aligned} \tag{A.4}$$

Using $w_D = 1 - w_S - w_G - w_M$, each dollar of government bonds reduces excess redemptions by $1 - \lambda/\left(1+\lambda\right)$ dollars. Reserves, on the other hand, reduce excess redemptions by $\overline{m} - \lambda/\left(1+\lambda\right)$. Therefore the liquidity services multiplier on reserves is

$$m = \frac{\overline{m} - \frac{\lambda}{1+\lambda}}{1 - \frac{\lambda}{1+\lambda}} = \overline{m} + \lambda\left(\overline{m} - 1\right). \tag{A.5}$$

As long as the liquidity premium on bonds is $1/m$ times the liquidity premium on reserves, which is itself equal to the nominal rate n, reserves and money are perfect substitutes. We can thus define the liquid asset portfolio share $w_L = w_G + mw_M$ and state the agent's optimization problem as in (15)–(16):

$$0 = \max_{c, w_S, w_L \geq 0} f\left(cW, V\left(W, \omega\right)\right)dt + E\left[dV\left(W, \omega\right)\right] \tag{A.6}$$

$$\begin{aligned}\frac{dW}{W} &= \left[r - c + w_S\left(\mu - r\right) - \frac{w_L}{m}n + \frac{\Pi}{m}n\right]dt + w_S\sigma dB \\ &\quad - \frac{\phi}{1-\phi}\max\left\{\frac{\lambda}{1+\lambda}\left(w_S + w_L - 1\right) - w_L, 0\right\}dN.\end{aligned} \tag{A.7}$$

Conditions for full self-insurance. From (20), the optimality condition for liquidity is

$$V_W^i W \frac{1}{m} n \;=\; \eta V_{W^+}^i \frac{\partial W^+}{\partial w_L} \;=\; \eta V_{W^+}^i W \frac{\phi}{1-\phi} \frac{1}{1+\lambda}. \tag{A.8}$$

The left side is the marginal cost of holding liquidity: the marginal value of wealth times the liquidity premium. The right side is the marginal benefit: the marginal value of wealth following a funding shock times the reduction in the expected fire sale losses a dollar of liquidity provides. Agents fully self-insure if benefits exceed costs. As V is concave in wealth (see Proposition 1), $V_{W^+}^i \geq V_W^i$. Thus a sufficient condition for full insurance is

$$\eta \frac{\phi}{1-\phi} \frac{\lambda}{1+\lambda} \;\geq\; \frac{\lambda}{m} n. \tag{A.9}$$

Since n is a function of ω, which is bounded, there is always a high enough ϕ or η to ensure (A.9) holds. Finally, from (16), fully meeting redemptions in case of a funding shocks requires holding liquidity in the amount given by (22).

Proof of Proposition 1. From (22), we can substitute $w_L = \max\{\lambda(w_S - 1), 0\}$ in the HJB equation (20). The non-negativity constraint on liquidity is already satisfied so it can be dropped. Also conjecture that V^i has the form in (23) and substitute for f^i from (3). Simplifying, wealth drops out and we get

$$\begin{aligned} 0 \;=\; \max_{c, w_S} & \left(\frac{1-\gamma^i}{1-1/\psi}\right)\left[\left(\frac{c}{(J^i)^{\frac{1}{1-\psi}}}\right)^{1-1/\psi} - (\rho+\kappa)\right] \\ & + \left(1-\gamma^i\right)\left[r - c + w_S(\mu - r) - \frac{\gamma^i}{2}(w_S\sigma)^2 - \frac{1}{m}\max\{\lambda(w_S-1), 0\} n + \frac{\Pi}{m} n\right] \\ & + \left(\frac{1-\gamma^i}{1-\psi}\right)\left[\frac{J_\omega^i}{J^i}\left[\kappa(\overline{\omega} - \omega) + \omega(1-\omega)\mu_\omega\right] + \left(1-\gamma^i\right)\frac{J_\omega^i}{J^i}\omega(1-\omega) w_S \sigma_\omega \sigma\right] \\ & + \frac{1}{2}\left(\frac{1-\gamma^i}{1-\psi}\right)\left[\left(\frac{1-\gamma^i}{1-\psi} - 1\right)\left(\frac{J_\omega^i}{J^i}\right)^2 + \frac{J_{\omega\omega}^i}{J^i}\right]\omega^2(1-\omega)^2\sigma_\omega^2. \end{aligned} \tag{A.10}$$

The FOC for consumption gives $c = J^i$ which can be substituted to simplify (A.10).

There are three possibilities for w_S: an interior optimum with $w_S > 1$, an interior optimum with $w_S < 1$, and a corner solution with $w_S = 1$. Let

$$\underline{w}_S \;=\; \frac{1}{\gamma^i}\left[\frac{\mu - r - (\lambda/m) n}{\sigma^2} + \left(\frac{1-\gamma}{1-\psi}\right)\frac{J_\omega^i}{J^i}\omega(1-\omega)\frac{\sigma_\omega}{\sigma}\right] \tag{A.11}$$

$$\overline{w}_S \;=\; \frac{1}{\gamma^i}\left[\frac{\mu - r}{\sigma^2} + \left(\frac{1-\gamma}{1-\psi}\right)\frac{J_\omega^i}{J^i}\omega(1-\omega)\frac{\sigma_\omega}{\sigma}\right]. \tag{A.12}$$

The three possibilities are

$$w_S = \begin{cases} \overline{w}_S & \text{if} \quad \overline{w}_S \leq 1 \\ 1 & \text{if} \quad \underline{w}_S \leq 1 < \overline{w}_S \\ \underline{w}_S & \text{if} \quad 1 < \underline{w}_S. \end{cases} \tag{A.13}$$

The market clearing equation (18) for the endowment claim implies that only one type of agents, if any, takes leverage, so the equilibrium must be in one of the three cases,

$$w_S^A > 1, \qquad w_S^B < 1 \tag{A.14}$$
$$w_S^A = 1, \qquad w_S^B = 1 \tag{A.15}$$
$$w_S^A < 1, \qquad w_S^B > 1. \tag{A.16}$$

Substituting,

$$\{w_S^A, w_S^B\} = \begin{cases} \{\underline{w}_S^A, \overline{w}_S^B\} & \text{if} \quad \overline{w}_S^B \leq 1 < \underline{w}_S^A \\ \{1, 1\} & \text{if} \quad \underline{w}_S^A, \underline{w}_S^B \leq 1 < \overline{w}_S^A, \overline{w}_S^B \\ \{\overline{w}_S^A, \underline{w}_S^B\} & \text{if} \quad \overline{w}_S^A \leq 1 < \underline{w}_S^B. \end{cases} \tag{A.17}$$

Call these three cases (i), (ii), and (iii). Under case (i),

$$w_S^A = \frac{1}{\gamma^A}\left[\frac{\mu - r - (\lambda/m)\, n}{\sigma^2} + \left(\frac{1-\gamma^A}{1-\psi}\right)\frac{J_\omega^A}{J^A}\omega\,(1-\omega)\left(\frac{\sigma_\omega}{\sigma}\right)\right] \tag{A.18}$$

$$w_S^B = \frac{1}{\gamma^B}\left[\frac{\mu - r}{\sigma^2} + \left(\frac{1-\gamma^B}{1-\psi}\right)\frac{J_\omega^B}{J^B}\omega\,(1-\omega)\left(\frac{\sigma_\omega}{\sigma}\right)\right]. \tag{A.19}$$

To get the dynamics of ω, apply Ito's lemma to (4) and use $W^A + W^B = P$ to obtain

$$\frac{d\omega}{\omega\,(1-\omega)} = \left(\frac{dW^A}{W^A} - \frac{dW^B}{W^B}\right) - \left(\frac{dW^A}{W^A} - \frac{dW^B}{W^B}\right)\left(\frac{dP}{P}\right). \tag{A.20}$$

Substituting for the evolution of aggregate type-A and type-B wealth gives

$$\mu_\omega = \left(w_S^A - w_S^B\right)(\mu - r) - \frac{\lambda}{m}\left(w_S^A - 1\right) n - \left(J^A - J^B\right) - \sigma_\omega \sigma \tag{A.21}$$

$$\sigma_\omega = \left(w_S^A - w_S^B\right)\sigma. \tag{A.22}$$

Note that by stock-market clearing (18)

$$\frac{\sigma_\omega}{\sigma} = w_S^A - w_S^B = \frac{1}{1-\omega}\left(w_S^A - 1\right). \tag{A.23}$$

From (A.3), return volatility is then

$$\sigma = \sigma_Y - \frac{F_\omega}{F}\omega\,(1-\omega)\,\sigma_\omega = \frac{\sigma_Y}{1 + \frac{F_\omega}{F}\omega\,(w_S^A - 1)}. \tag{A.24}$$

From the goods-market clearing condition (17), the dividend yield is $F = \omega J^A + (1-\omega) J^B$. Expressing w_S^B in terms of w^A, stock-market clearing (18) gives

$$
\begin{aligned}
1 &= \omega w_S^A + (1-\omega) w_S^B \qquad (A.25)\\
&= \omega w_S^A + (1-\omega)\frac{1}{\gamma^B}\left\{\gamma^A w_S^A + \left(\frac{\lambda}{m}n\right)\frac{\left[1+\frac{F_\omega}{F}\omega\left(w_S^A-1\right)\right]^2}{\sigma_Y^2}\right. \qquad (A.26)\\
&\quad \left. - \left[\left(\frac{1-\gamma^A}{1-\psi}\right)\frac{J_\omega^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi}\right)\frac{J_\omega^B}{J^B}\right]\omega\left(w_S^A-1\right)\right\}.
\end{aligned}
$$

This gives a quadratic equation for w_S^A in terms of exogenous quantities and the conjectured J^A and J^B. The solution is the positive root. From here w_S^B follows from stock-market clearing. We need to verify $w_S^A > 1$. Plugging $w_S^A = 1$ into (A.26) shows that this requires

$$
\frac{\lambda}{m}n \;<\; \left(\gamma^B - \gamma^A\right)\sigma_Y^2. \qquad (A.27)
$$

From w_S^A we can get $(\mu - r)/\sigma^2$, and σ_ω/σ. This also gives σ and hence σ_ω, and as a result, $\mu - r$ and μ_ω from (A.21). We can then get μ from (A.2), which gives $r = \mu - (\mu - r)$. Finally, plug $w_S^A = \underline{w}_S^A$ and $w_S^B = \overline{w}_S^B$ into the HJB equations to verify the conjectures for J^A and J^B. For the value of liquidity, use the liquid-asset market clearing equation (19),

$$
\left\{w_L^A, w_L^B\right\} \;=\; \left\{\frac{\Pi}{\omega}, 0\right\}. \qquad (A.28)
$$

The binding leverage constraint pins down the value of liquidity:

$$
\Pi \;=\; \omega\lambda\left(w_S^A - 1\right). \qquad (A.29)
$$

Under case (ii), $\left\{w_S^A, w_S^B\right\} = \{1,1\}$ and $\left\{w_L^A, w_L^B\right\} = \{0,0\}$. The stock market clears and $\Pi = 0$. From here, we get $\sigma_\omega = 0$ and so $\sigma = \sigma_Y$. Next, use

$$
\mu_\omega \;=\; -\left(J^A - J^B\right) \qquad (A.30)
$$

in the dynamics of returns (A.2) and (A.3) to get μ and σ. Substituting into the HJB equations and simplifying,

$$
\rho + \kappa \;=\; 1/\psi J^i + (1 - 1/\psi)\left(\mu - \frac{\gamma^i}{2}\sigma_Y^2\right) - 1/\psi \frac{J_\omega^i}{J^i}\left[\kappa\left(\overline{\omega} - \omega\right) + \omega(1-\omega)\mu_\omega\right]. \qquad (A.31)
$$

This case requires $(\lambda/m)\,n > \left|\gamma^A - \gamma^B\right|\sigma_Y^2$ (there is no excess volatility because $\sigma_\omega = 0$. The real interest rate lies inside a range between a lending and a borrowing rate.

Case (iii) is analogous to case (i) with the roles reversed. It requires $(\lambda/m)\,n < \left(\gamma^A - \gamma^B\right)\sigma_Y^2$, which is ruled out by $\gamma^B > \gamma^A$. This completes the proof. □

Proof of Proposition 2. A agents take leverage in case (i) in the proof of Proposition 1. From (A.27), this requires $(\lambda/m)\,n < \left(\gamma^B - \gamma^A\right)\sigma_Y^2$. A agents' demand w_S^A is as in (A.18). □

Proof of Proposition 3. The value of Π_t follows from the fact that A agents, if anyone, use leverage, and that they buy liquidity insurance to the point of full insurance (see (A.29)). To obtain (28), apply Ito's Lemma to $\Pi_t P_t = \pi_t G_t + (m-1)\,\pi_t M_t$ (see (8)) and use the fact that inflation $-d\pi_t/\pi_t = \iota_t dt = (n_t - r_t)\,dt$ is locally deterministic (see (9)). $\square$

B. *Extended model*

This Appendix contains derivations for the extended model in Section VI.

Adjustment cost function. We consider a specification for ϕ consistent with quadratic investment adjustment costs (see Brunnermeier and Sannikov, 2014a):

$$\phi(i) \;=\; \frac{1}{\varphi}\left(\sqrt{1+2\varphi i}-1\right). \tag{B.1}$$

Using (38), this gives optimal investment $i = \frac{1}{2\varphi}\left(q^2-1\right)$ and $\phi(i) = \frac{1}{\varphi}(q-1)$. The expected return is $\mu = \left(a + 1/(2\varphi)\right)/q + q/(2\varphi) - 1/\varphi - \delta + \mu_q + \sigma_q\sigma_k$. Substituting into (39) for i, using $c^A = J^A$ and $c^B = J^B$ (see below), and solving for q (the positive root),

$$q \;=\; \varphi\left(-\left[\omega J^A + (1-\omega)\,J^B\right] + \sqrt{\left[\omega J^A + (1-\omega_t)\,J^B\right]^2 + \frac{2}{\varphi}\left(a + o + \frac{1}{2\varphi}\right)}\right). \tag{B.2}$$

Denote the dynamics of ω by

$$d\omega \;=\; \kappa(\overline{\omega}-\omega)\,dt + \omega(1-\omega)\left[\mu_\omega(\omega,n)\,dt + \sigma_\omega(\omega,n)'\,dB\right]. \tag{B.3}$$

Applying Ito's Lemma, the dynamics of q are

$$\begin{aligned}
\mu_q \;&=\; \frac{q_\omega}{q}\left[\kappa(\overline{\omega}-\omega) + \omega(1-\omega)\,\mu_\omega\right] - \frac{q_n}{q}\kappa_n(n-n_0) + \frac{1}{2}\frac{q_{\omega\omega}}{q}\omega^2(1-\omega)^2\sigma_\omega'\sigma_\omega \qquad \text{(B.4)}\\
&\quad + \frac{q_{\omega n}}{q}\omega(1-\omega)\sqrt{(n-\underline{n})(\overline{n}-n)}\sigma_\omega'\begin{bmatrix}0\\ \sigma_n\end{bmatrix} + \frac{q_{nn}}{q}(n-\underline{n})(\overline{n}-n)\,\sigma_n^2\\
\sigma_q \;&=\; \frac{q_\omega}{q}\omega(1-\omega)\,\sigma_\omega + \frac{q_n}{q}\sqrt{(n-\underline{n})(\overline{n}-n)}\begin{bmatrix}0\\ \sigma_n\end{bmatrix}. \qquad \text{(B.5)}
\end{aligned}$$

These can be plugged into (36) to obtain expressions for μ and σ.

Proposition B.1. *Under the extended model, the value function of an agent of type $i \in \{A, B\}$ has the form*

$$V^i(W,\omega,n) \;=\; \left(\frac{W^{1-\gamma}}{1-\gamma}\right)J^i(\omega,n)^{\frac{1-\gamma}{1-\psi}}, \tag{B.6}$$

where $J^i(\omega,n)$ represents the agents' optimal consumption-wealth ratio, $c = J^i$. A agents *(banks) take leverage ($w_S^A > 1$) if and only if*

$$\begin{aligned}\frac{\lambda}{m}n &< \left(\gamma^B-\gamma^A\right)\left[\sigma_k^2+\left(\frac{q_n}{q}\right)^2\left(n-\underline{n}\right)\left(\overline{n}-n\right)\sigma_n^2\right] \\ &\quad +\left[\left(\frac{1-\gamma^A}{1-\psi}\right)\frac{J_n^A}{J^A}-\left(\frac{1-\gamma^B}{1-\psi}\right)\frac{J_n^B}{J^B}\right]\left[\frac{q_n}{q}\left(n-\underline{n}\right)\left(\overline{n}-n\right)\sigma_n^2\right].\end{aligned} \tag{B.7}$$

In this case, their holdings of the risky asset are given by

$$\begin{aligned}w_S^A &= \frac{1}{\gamma^A}\left\{\frac{\mu-r-(\lambda/m)\,n}{\sigma'\sigma}\right. \\ &\quad \left.+\left(\frac{1-\gamma^A}{1-\psi}\right)\left[\frac{J_\omega^A}{J^A}\omega\left(1-\omega\right)\left(\frac{\sigma_\omega'\sigma}{\sigma'\sigma}\right)+\frac{J_n^A}{J^A}\sqrt{\left(n-\underline{n}\right)\left(\overline{n}-n\right)}\left(\frac{\sigma'}{\sigma'\sigma}\begin{bmatrix}0\\ \sigma_n\end{bmatrix}\right)\right]\right\}.\end{aligned} \tag{B.8}$$

Proof of Proposition B.1. In the extended model, the HJB equation (20) becomes

$$\begin{aligned}0 &= \max_{c,i,w_S,w_L\geq 0} f\left(cW,V\right)+V_W W\left[r+\frac{o}{q}-c+w_S\left(\mu-r\right)-\frac{w_L}{m}n+\frac{\Pi}{m}n\right] \\ &\quad +V_\omega\left[\kappa\left(\overline{\varpi}-\omega\right)+\omega\left(1-\omega\right)\mu_\omega\right]+V_n\kappa_n\left(n-n_b\right)+V_{W\omega}W\omega\left(1-\omega\right)w_S\sigma_\omega'\sigma \\ &\quad +V_{Wn}Ww_S\sqrt{\left(n-\underline{n}\right)\left(\overline{n}-n\right)}\sigma'\begin{bmatrix}0\\ \sigma_n\end{bmatrix}+\frac{1}{2}V_{WW}W^2w_S^2\sigma'\sigma+\frac{1}{2}V_{\omega\omega}\omega^2\left(1-\omega\right)^2\sigma_\omega'\sigma_\omega \\ &\quad +V_{\omega n}\omega\left(1-\omega\right)+\sqrt{\left(n-\underline{n}\right)\left(\overline{n}-n\right)}\sigma_\omega'\begin{bmatrix}0\\ \sigma_n\end{bmatrix}+\frac{1}{2}V_{nn}\left(n-\underline{n}\right)\left(\overline{n}-n\right)\sigma_n^2 \\ &\quad +\eta\left(V_+-V\right).\end{aligned} \tag{B.9}$$

The full-insurance liquidity demand is the same as before (see (21)). When it holds $V_+-V=0$. Plugging into the HJB equation,

$$\begin{aligned}0 &= \max_{c,i,w_S} f\left(cW,V\right)+V_W W\left[r+\frac{o}{q}-c+w_S\left(\mu-r\right)-\frac{1}{m}\max\left[\lambda\left(w_S-1\right),0\right]n+\frac{\Pi}{m}n\right] \\ &\quad +V_\omega\left[\kappa\left(\overline{\varpi}-\omega\right)+\omega\left(1-\omega\right)\mu_\omega\right]+V_n\kappa_n\left(n-n_b\right)+V_{W\omega}W\omega\left(1-\omega\right)w_S\sigma_\omega'\sigma \\ &\quad +V_{Wn}Ww_S\sqrt{\left(n-\underline{n}\right)\left(\overline{n}-n\right)}\sigma'\begin{bmatrix}0\\ \sigma_n\end{bmatrix}+\frac{1}{2}V_{WW}W^2w_S^2\sigma'\sigma+\frac{1}{2}V_{\omega\omega}\omega^2\left(1-\omega\right)^2\sigma_\omega'\sigma_\omega \\ &\quad +V_{\omega n}\omega\left(1-\omega\right)\sqrt{\left(n-\underline{n}\right)\left(\overline{n}-n\right)}\sigma_\omega'\begin{bmatrix}0\\ \sigma_n\end{bmatrix}+\frac{1}{2}V_{nn}\left(n-\underline{n}\right)\left(\overline{n}-n\right)\sigma_n^2.\end{aligned} \tag{B.10}$$

As shown in (38), the optimal investment policy satisfies $\phi'(i)\,q=1$. From here on we evaluate the return on capital μ at this optimal level of investment.

Conjecture that the value function has the form

$$V\left(W,\omega,n\right) = \left(\frac{W^{1-\gamma}}{1-\gamma}\right)J\left(\omega,n\right)^{\frac{1-\gamma}{1-\psi}}. \tag{B.11}$$

Then wealth drops out of the HJB equation:

$$
\begin{aligned}
0 \;=\; & \max_{c,w_S}\; c^{1-1/\psi}J^{1/\psi}-(\rho+\kappa)+(1-1/\psi)\left[r+\frac{o}{q}-c+w_S(\mu-r)+\frac{\Pi}{m}n-\frac{\gamma}{2}w_S^2\sigma'\sigma\right. \\
& \left.-\frac{1}{m}\max\left[\lambda(w_S-1),0\right]n\right]-1/\psi\left[\frac{J_\omega}{J}\left[\kappa(\overline{\omega}-\omega)+\omega(1-\omega)\mu_\omega\right]+\frac{J_n}{J}\kappa_n(n-n_b)\right. \\
& \left.+(1-\gamma)\frac{J_\omega}{J}\omega(1-\omega)w_S\sigma_\omega'\sigma+(1-\gamma)\frac{J_n}{J}w_S\sqrt{(n-\underline{n})(\overline{n}-n)}\sigma'\begin{bmatrix}0\\ \sigma_n\end{bmatrix}\right] \\
& -\frac{1/\psi}{2}\left[\left(\frac{1-\gamma}{1-\psi}-1\right)\left(\frac{J_\omega}{J}\right)^2+\frac{J_{\omega\omega}}{J}\right]\omega^2(1-\omega)^2\sigma_\omega'\sigma_\omega \\
& -1/\psi\left[\left(\frac{1-\gamma}{1-\psi}-1\right)\left(\frac{J_\omega}{J}\right)\left(\frac{J_n}{J}\right)+\frac{J_{\omega n}}{J}\right]\omega(1-\omega)\sqrt{(n-\underline{n})(\overline{n}-n)}\sigma_\omega'\begin{bmatrix}0\\ \sigma_n\end{bmatrix} \\
& -\frac{1/\psi}{2}\left[\left(\frac{1-\gamma}{1-\psi}-1\right)\left(\frac{J_n}{J}\right)^2+\frac{J_{nn}}{J}\right](n-\underline{n})(\overline{n}-n)\sigma_n^2. \qquad \text{(B.12)}
\end{aligned}
$$

The FOC for consumption gives $c = J$. Let

$$
\begin{aligned}
\underline{w}_S \;=\; & \frac{1}{\gamma}\left\{\frac{\mu-r-(\lambda/m)n}{\sigma'\sigma}\right. \qquad \text{(B.13)} \\
& \left.+\left(\frac{1-\gamma}{1-\psi}\right)\left[\frac{J_\omega}{J}\omega(1-\omega)\left(\frac{\sigma_\omega'\sigma}{\sigma'\sigma}\right)+\frac{J_n}{J}\sqrt{(n-\underline{n})(\overline{n}-n)}\left(\frac{\sigma'\begin{bmatrix}0\\ \sigma_n\end{bmatrix}}{\sigma'\sigma}\right)\right]\right\} \\
\overline{w}_S \;=\; & \frac{1}{\gamma}\left\{\frac{\mu-r}{\sigma'\sigma}\right. \qquad \text{(B.14)} \\
& \left.+\left(\frac{1-\gamma}{1-\psi}\right)\left[\frac{J_\omega}{J}\omega(1-\omega)\left(\frac{\sigma_\omega'\sigma}{\sigma'\sigma}\right)+\frac{J_n}{J}\sqrt{(n-\underline{n})(\overline{n}-n)}\left(\frac{\sigma'\begin{bmatrix}0\\ \sigma_n\end{bmatrix}}{\sigma'\sigma}\right)\right]\right\}.
\end{aligned}
$$

There are three possible cases:

$$
w_S = \begin{cases} \overline{w}_S & \text{if} \quad \overline{w}_S \le 1 \\ 1 & \text{if} \quad \underline{w}_S \le 1 < \overline{w}_S \\ \underline{w}_S & \text{if} \quad 1 < \underline{w}_S. \end{cases} \qquad \text{(B.15)}
$$

Market clearing implies that only one type of agent, at most, takes leverage, so there are three possible cases in equilibrium:

$$
\begin{aligned}
w_S^A > 1, &\quad w_S^B < 1 \qquad \text{(B.16)} \\
w_S^A = 1, &\quad w_S^B = 1 \qquad \text{(B.17)} \\
w_S^A < 1, &\quad w_S^B > 1. \qquad \text{(B.18)}
\end{aligned}
$$

Call these three cases (i), (ii), and (iii). Under case (i), $w_S^A = \underline{w}_S^A$ and $w_S^B = \overline{w}_S^B$. Using (A.23) and (B.5), we can simplify σ in (36) as follows:

$$\sigma = \left[1 - \frac{q_\omega}{q}\omega\left(w_S^A - 1\right)\right]^{-1}\left(\begin{bmatrix}\sigma_k \\ 0\end{bmatrix} + \frac{q_n}{q}\sqrt{\left(n - \underline{n}\right)\left(\overline{n} - n\right)}\begin{bmatrix}0 \\ \sigma_n\end{bmatrix}\right). \tag{B.19}$$

From (B.5) and (B.19),

$$\frac{\sigma_\omega'\sigma}{\sigma'\sigma} = \frac{1}{1-\omega}\left(w_S^A - 1\right) \tag{B.20}$$

$$\frac{\sigma'\begin{bmatrix}0 \\ \sigma_n\end{bmatrix}}{\sigma'\sigma} = \left[1 - \frac{q_\omega}{q}\omega\left(w_S^A - 1\right)\right]\frac{\frac{q_n}{q}\sqrt{\left(n - \underline{n}\right)\left(\overline{n} - n\right)}\sigma_n^2}{\sigma_k^2 + \left(\frac{q_n}{q}\right)^2\left(n - \underline{n}\right)\left(\overline{n} - n\right)\sigma_n^2}. \tag{B.21}$$

Expressing w_S^B in terms of w^A, stock-market clearing (18) gives

$$1 = \omega w_S^A + (1-\omega)\, w_S^B \tag{B.22}$$

$$\begin{aligned} &= \omega w_S^A + (1-\omega)\frac{1}{\gamma^B}\left\{\gamma^A w_S^A + \left(\frac{\lambda}{m}n\right)\frac{\left[1 - \frac{q_\omega}{q}\omega\left(w_S^A - 1\right)\right]^2}{\sigma_k^2 + \left(\frac{q_n}{q}\right)^2\left(n - \underline{n}\right)\left(\overline{n} - n\right)\sigma_n^2}\right. \\ &\quad - \left[\left(\frac{1-\gamma^A}{1-\psi}\right)\frac{J_\omega^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi}\right)\frac{J_\omega^B}{J^B}\right]\omega\left(w_S^A - 1\right) - \left[1 - \frac{q_\omega}{q}\omega\left(w_S^A - 1\right)\right] \\ &\quad \left.\cdot\left[\left(\frac{1-\gamma^A}{1-\psi}\right)\frac{J_n^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi}\right)\frac{J_n^B}{J^B}\right]\left[\frac{\frac{q_n}{q}\left(n - \underline{n}\right)\left(\overline{n} - n\right)\sigma_n^2}{\sigma_k^2 + \left(\frac{q_n}{q}\right)^2\left(n - \underline{n}\right)\left(\overline{n} - n\right)\sigma_n^2}\right]\right\}. \end{aligned} \tag{B.23}$$

This gives a quadratic equation for w_S^A in terms of exogenous quantities and the conjectured J^A, J^B, and o (q is given by (B.2)). The solution is the positive root. From here w_S^B follows from stock-market clearing. We need to verify $w_S^A > 1$, which requires

$$\begin{aligned} \frac{\lambda}{m}n &< \left(\gamma^B - \gamma^A\right)\left[\sigma_k^2 + \left(\frac{q_n}{q}\right)^2\left(n - \underline{n}\right)\left(\overline{n} - n\right)\sigma_n^2\right] \\ &\quad + \left[\left(\frac{1-\gamma^A}{1-\psi}\right)\frac{J_n^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi}\right)\frac{J_n^B}{J^B}\right]\left[\frac{q_n}{q}\left(n - \underline{n}\right)\left(\overline{n} - n\right)\sigma_n^2\right]. \end{aligned} \tag{B.24}$$

The first term on the right is once again $\left(\gamma^B - \gamma^A\right)\sigma'\sigma$ as in the baseline model, while the second term takes into account differences in n-hedging demand that persist in autarky.

From w_S^A we can get $(\mu - r)/(\sigma'\sigma)$, $(\sigma_\omega'\sigma)/(\sigma'\sigma)$, and $\left(\sigma'\begin{bmatrix}0 \\ \sigma_n\end{bmatrix}\right)/(\sigma'\sigma)$. This also gives σ and hence σ_ω, and as a result, $\mu - r$. Next, calculate w_S^B and

$$\mu_\omega = \left(w_S^A - w_S^B\right)(\mu - r) - \frac{\lambda}{m}\left(w_S^A - 1\right)n - \left(J^A - J^B\right) - \sigma_\omega'\sigma. \tag{B.25}$$

This can be plugged into the dynamics of returns to get μ. Finally plug into the two HJB equations and $r = \overline{r}$ to solve for J^A, J^B, and l. The liquid asset market must clear:

$$\left\{w_L^A, w_L^B\right\} = \left\{\frac{\Pi}{\omega}, 0\right\}. \tag{B.26}$$

A agents' liquidity demand pins down the liquidity supply $\Pi = \omega\lambda\left(w_S^A - 1\right)$.

Under case (ii), $\left\{w_S^A, w_S^B\right\} = \{1,1\}$ and $\left\{w_L^A, w_L^B\right\} = \{0,0\}$. The stock market clears and $\Pi = 0$. From here, we get $\sigma_\omega = 0$ and so $\sigma = \sigma_k + \frac{q_n}{q}\sqrt{(n-\underline{n})(\overline{n}-n)}\sigma_n$. Next, use $\mu_\omega = -\left(J^A - J^B\right)$ in the dynamics of returns to get μ and σ. Substituting into the HJB equations and simplifying,

$$\begin{aligned}\rho+\kappa &= 1/\psi J + (1-1/\psi)\left[\mu + \frac{o}{q} + \left(\frac{1-\gamma}{1-\psi}\right)\frac{J_n}{J}\sqrt{(n-\underline{n})(\overline{n}-n)}\sigma'\begin{bmatrix}0\\ \sigma_n\end{bmatrix} - \frac{\gamma}{2}\sigma'\sigma\right] \\ &\quad -1/\psi\left[\frac{J_\omega}{J}\left[\kappa\left(\overline{\omega}-\omega\right)+\omega\left(1-\omega\right)\mu_\omega\right] + \frac{J_n}{J}\kappa_n\left(n-n_b\right)\right] \\ &\quad -\frac{1/\psi}{2}\left[\left(\frac{\psi-\gamma}{1-\psi}\right)\left(\frac{J_n}{J}\right)^2 + \frac{J_{nn}}{J}\right](n-\underline{n})(\overline{n}-n)\sigma_n^2. \end{aligned} \tag{B.27}$$

This case requires

$$\begin{aligned}\frac{\lambda}{m}n &> \left|\left(\gamma^B-\gamma^A\right)\left[\sigma_k^2 + \left(\frac{q_n}{q}\right)^2(n-\underline{n})(\overline{n}-n)\sigma_n^2\right]\right. \\ &\quad \left. + \left[\left(\frac{1-\gamma^A}{1-\psi}\right)\frac{J_n^A}{J^A} - \left(\frac{1-\gamma^B}{1-\psi}\right)\frac{J_n^B}{J^B}\right]\left[\frac{q_n}{q}(n-\underline{n})(\overline{n}-n)\sigma_n^2\right]\right|. \end{aligned} \tag{B.28}$$

The real interest rate lies inside a range between a lending and a borrowing rate.

Case (iii) is analogous to Case (i) with the roles of A and B agents reversed. This completes the proof. $\square$

We solve the model using Chebyshev collocation with complete polynomials up to order N in both ω and n with $N = 30$.

The term structure of interest rates. Consider a zero-coupon nominal bond with maturity τ and face value one. The bond is issued at $t = 0$ and pays π_τ units of consumption at $t = \tau$, where we have normalized the initial inverse price level to $\pi_0 = 1$. Let $p_t^\tau = p^\tau(t, \omega_t, n_t)$ be the nominal shadow price of the bond from the perspective of B agents who are unconstrained (the real shadow price is $\pi_t p_t^\tau$). The stochastic discount factor of B agents is their marginal utility of wealth $V_W = W^{-\gamma}\left(J^B\right)^{\frac{1-\gamma}{1-\psi}}$. Then p_t^τ solves the terminal value problem

$$E_t\left[\frac{dp_t^\tau}{p_t^\tau} - \iota_t dt\right] - r_t dt = -E_t\left[\left(-\gamma\frac{dW}{W} + \frac{\gamma-1}{\psi-1}\frac{dJ_t^B}{J_t^B}\right)\left(\frac{dp_t^\tau}{p_t^\tau} - \iota_t dt\right)\right] \tag{B.29}$$

$$p_\tau^\tau = 1. \tag{B.30}$$

The pricing equation can equivalently be derived by solving B agents' optimization problem for the optimal portfolio weight for the nominal bond and setting that weight to zero. Using $n_t = r_t + \iota_t$, applying Ito's Lemma and rearranging,

$$\begin{aligned} n_t \;=\; & \frac{\partial p_t^\tau/\partial t}{p_t^\tau} + \frac{\partial p_t^\tau/\partial \omega}{p_t^\tau}\left[\kappa\left(\overline{\omega}-\omega_t\right)+\omega_t\left(1-\omega_t\right)\mu_{\omega,t}\right] - \frac{\partial p_t^\tau/\partial n}{p_t^\tau}\kappa_n\left(n_t - n_{0,t}\right) \\ & + \frac{1}{2}\frac{\partial^2 p_t^\tau/\partial \omega^2}{p}\omega_t^2\left(1-\omega_t\right)^2\sigma_{\omega,t}'\sigma_{\omega,t} + \frac{1}{2}\frac{\partial^2 p_t^\tau/\partial n^2}{p_t^\tau}\left(n_t-\underline{n}\right)\left(\overline{n}-n_t\right)\sigma_n^2 \\ & + \frac{\partial^2 p_t^\tau/\partial\omega\partial n}{p_t^\tau}\omega_t\left(1-\omega_t\right)\sqrt{\left(n_t-\underline{n}\right)\left(\overline{n}-n_t\right)}\sigma_{\omega,t}'\begin{bmatrix}0\\ \sigma_n\end{bmatrix} \\ & -\gamma w_s^B\sigma'\left[\frac{\partial p_t^\tau/\partial\omega}{p}\omega_t\left(1-\omega_t\right)\sigma_{\omega,t} + \frac{\partial p_t^\tau/\partial n}{p_t^\tau}\sqrt{\left(n_t-\underline{n}\right)\left(\overline{n}-n_t\right)}\begin{bmatrix}0\\ \sigma_n\end{bmatrix}\right] \\ & + \left(\frac{\gamma-1}{\psi-1}\right)\left[\frac{\partial J_t^B/\partial\omega}{J_t^B}\frac{\partial p_t^\tau/\partial\omega}{p_t^\tau}\omega_t^2\left(1-\omega_t\right)^2\sigma_{\omega,t}'\sigma_{\omega,t}\right. \\ & + \left(\frac{\partial J_t^B/\partial\omega}{J_t^B}\frac{\partial p_t^\tau/\partial n}{p_t^\tau} + \frac{\partial J_t^B/\partial n}{J_t^B}\frac{\partial p_t^\tau/\partial\omega}{p_t^\tau}\right)\omega_t\left(1-\omega_t\right)\sqrt{\left(n_t-\underline{n}\right)\left(\overline{n}-n_t\right)}\sigma_{\omega,t}'\begin{bmatrix}0\\ \sigma_n\end{bmatrix} \\ & \left. + \frac{\partial J_t^B/\partial n}{J_t^B}\frac{\partial p_t^\tau/\partial n}{p_t^\tau}\left(n_t-\underline{n}\right)\left(\overline{n}-n_t\right)\sigma_n^2\right]. \end{aligned} \tag{B.31}$$

The nominal yield is $y_t^\tau = -\left(1/\tau\right)\log p_t^\tau$. We solve this equation backwards from $t=\tau$.

Description	Parameter	Value
Risk aversion of type A	γ^A	1.5
Risk aversion of type B	γ^B	15
Elasticity of intertemporal substitution (EIS)	ψ	3
Rate of time preference	ρ	0.01
Agent death rate	κ	0.01
Population share of type A	$\overline{\omega}$	0.1
Endowment growth rate	μ_Y	0.02
Endowment volatility	σ_Y	0.02
Funding shock size	$\lambda/(1+\lambda)$	0.29
Funding shock frequency	η	0.10
Fire sale loss	ϕ	0.15
Government bond liquidity services ratio	$1/m$	0.25
Nominal rate policy 1	n_1	0%
Nominal rate policy 2	n_2	5%

Table I: **Parameter values.** This table lists the parameter values used to illustrate the results of the model.

Description	Parameter	Value
Productivity of capital	a	0.03
Capital depreciation rate	δ	0.0075
Adjustment cost parameter	φ	1
Volatility of cash flows	σ_k	0.02
Nominal rate lower bound	$\underline{n}$	0.00
Nominal rate upper bound	$\underline{n}$	0.05
Benchmark policy rule	$n_0(\omega_t)$	0.01
Mean-reversion	κ_n	0.2
Volatility parameter	σ_n	0.5

Table II: **Parameter values for the extended model.** This table lists the parameter values used to illustrate the results of the extended model. The investment adjustment cost function is parameterized as $\phi(i_t) = 1/\varphi\left(\sqrt{1+2\varphi i_t}-1\right)$. The endowment process parameters μ_Y and σ_Y are replaced by the technology parameters a, δ, φ, and σ_k. All remaining parameters are as in Table I.

Figure 1: **The Fed Funds-T-Bill spread and the Fed Funds rate.** The figure plots the spread between the fed funds rate and the 3-Month T-bill (solid red, left axis), and the fed funds rate (dashed black, right axis). Monthly data, 1955 to 2010.

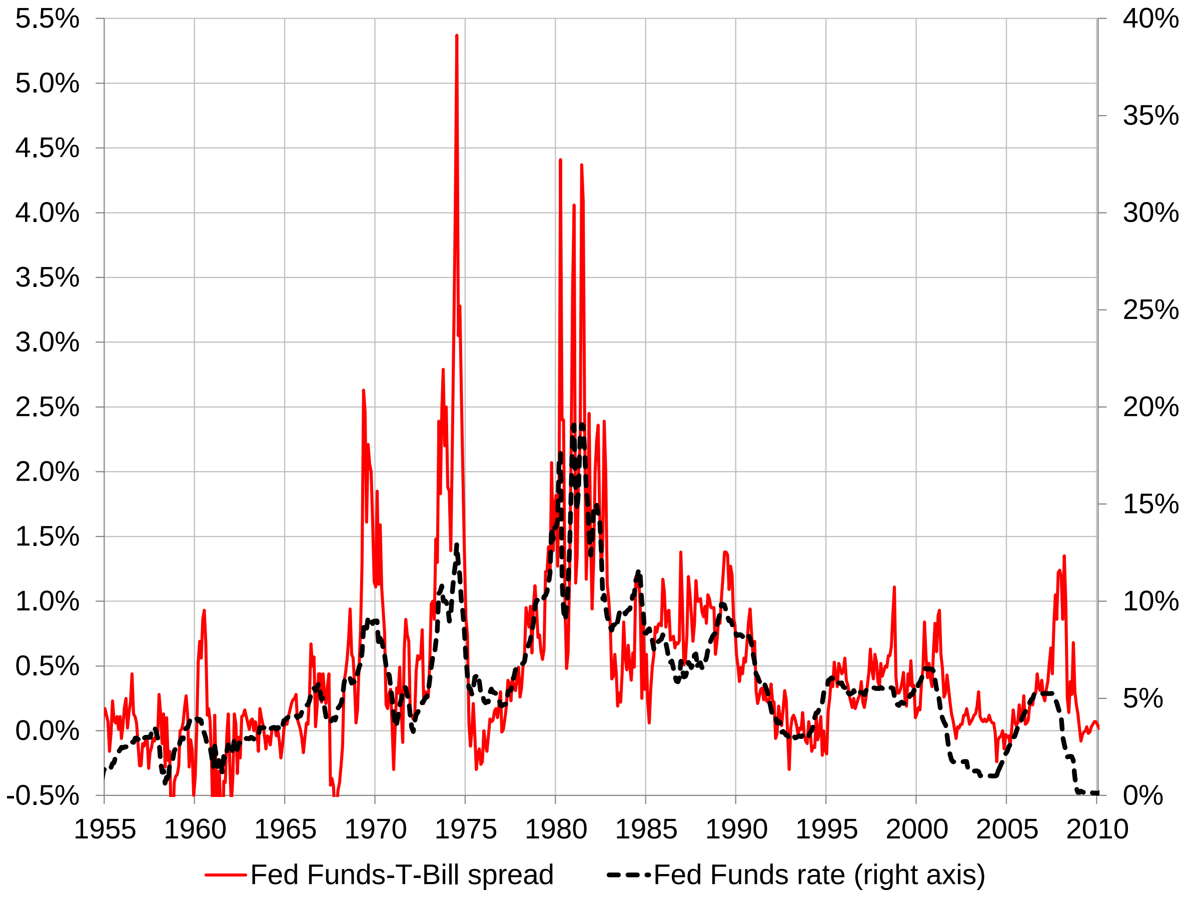

Figure 2: **Risk taking, risk premia, and the price of risk**
The figure plots the portfolio weight in the risky asset w_S for A agents (banks) and B agents (depositors), and the risk premium $\mu - r$ and Sharpe ratio $(\mu - r)/\sigma$ of the risky asset. Each line corresponds to a nominal rate policy: $n_l = 0\%$ (solid red) and $n_h = 5\%$ (dashed black).

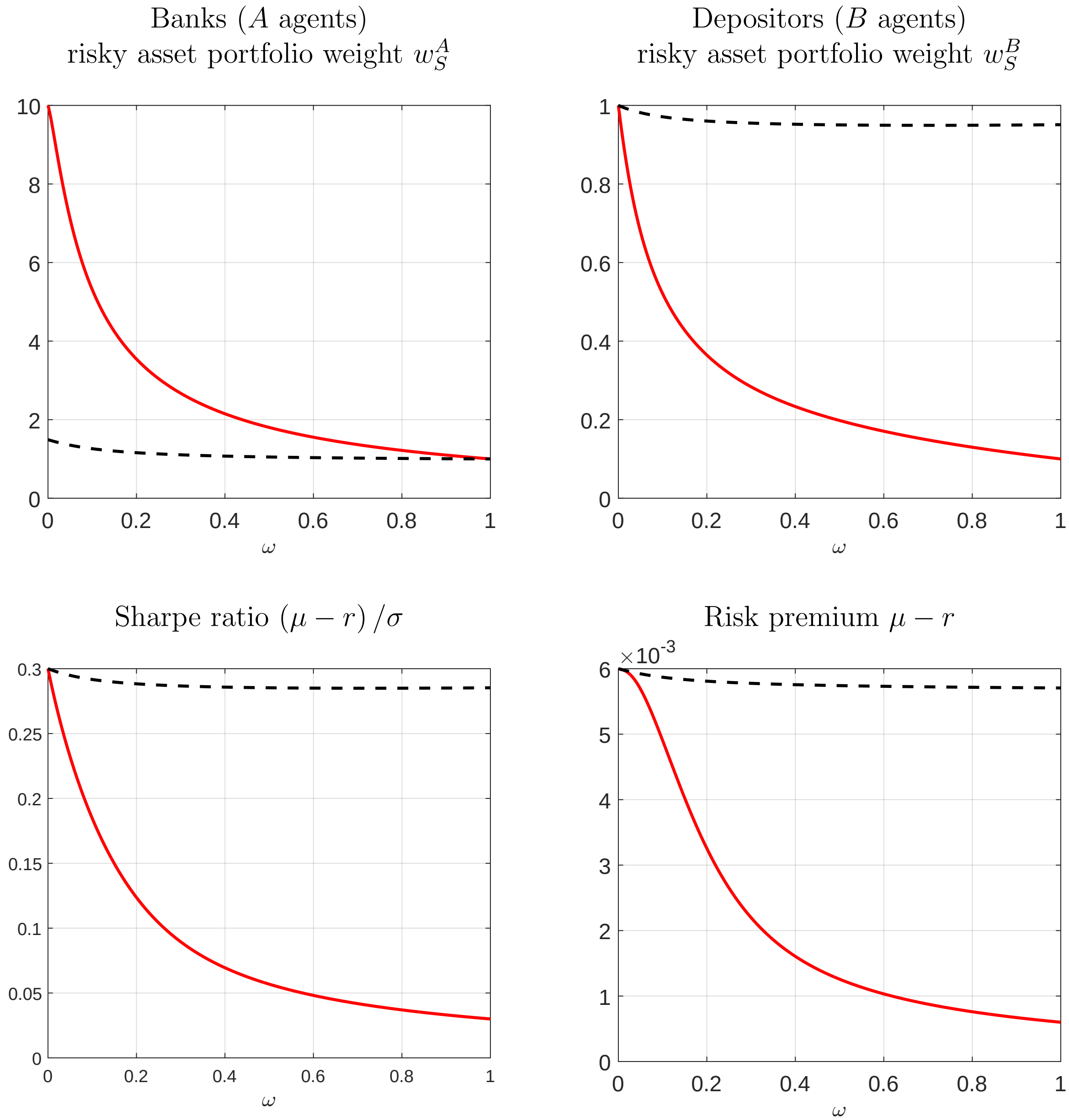

Figure 3: **The real rate, valuations, volatility, and bank net worth**
The figure plots the real interest rate r, the valuation ratio P/Y and volatility σ of the risky asset, and the stationary density of ω, the wealth share of A agents (banks). The stationary density is obtained by solving the associated forward Kolmogorov equation. Each line corresponds to a nominal rate policy: $n_l = 0\%$ (solid red lines) and $n_h = 5\%$ (dashed black lines).

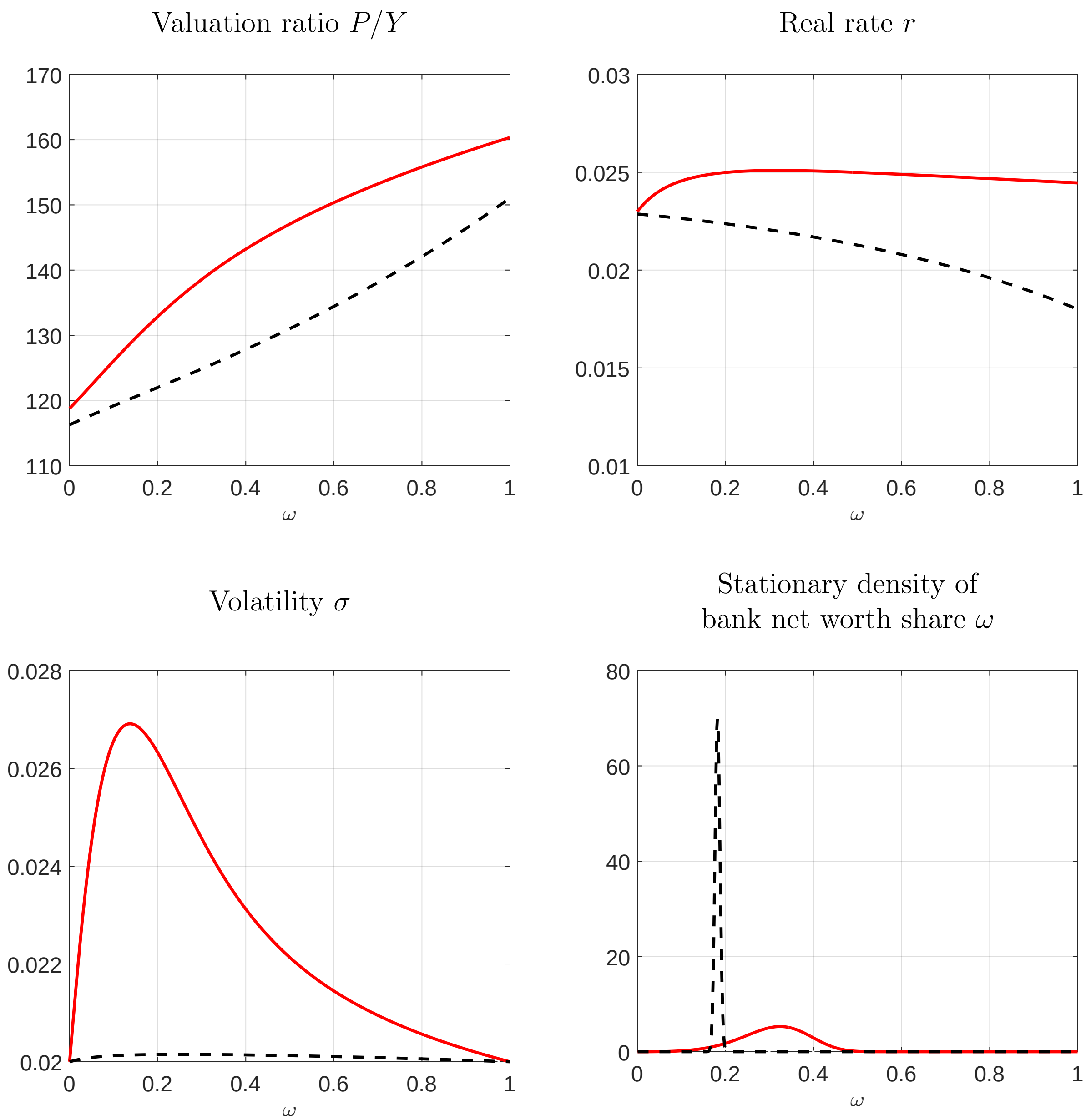

Figure 4: **Aggregate liquidity and open market operations**
The figure plots the liquidity supply as a share of aggregate wealth Π, the liquidity supply as a share of bank assets $\Pi/\left(\omega w_S^A\right)$, and the drift rate μ_M and stochastic exposure σ_M of open market operation (reserves growth) required to implement a given nominal rate policy. To calculate open market operations, we assume government bonds grow with total wealth ($dG/G = \iota dt + dP/P$) and that reserves represent 14% of the liquidity-services-adjusted liquidity supply ($\alpha = 0.14$). See the text for explanation. Each line corresponds to a nominal rate policy: $n_l = 0\%$ (solid red lines) and $n_h = 5\%$ (dashed black lines).

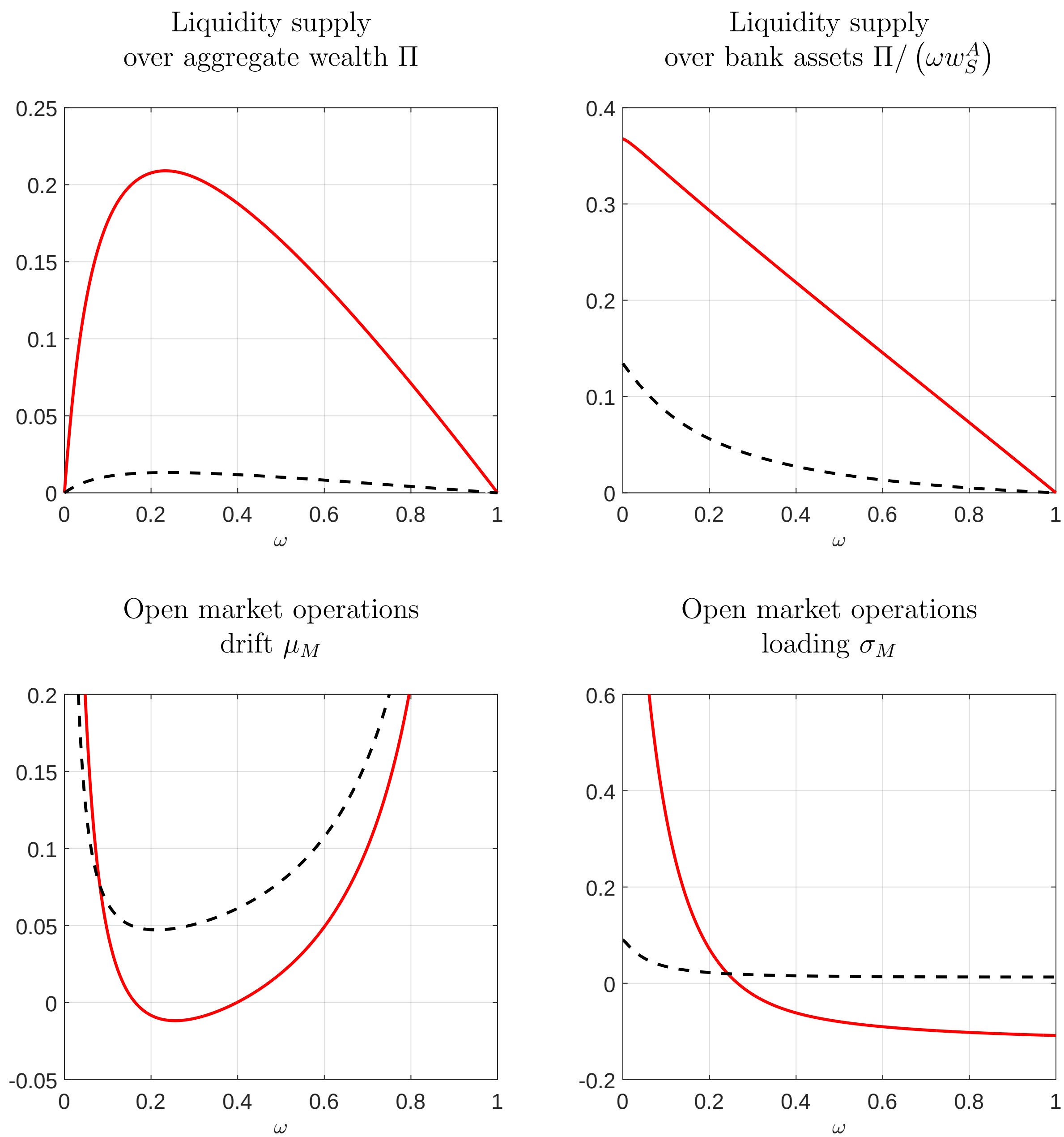

Figure 5: **Forward guidance and asset prices**
The figure plots the impact of forward guidance on asset prices. The left panel plots the two nominal rate policies n_0 (dashed black line) and n_{fg} (solid red line). The right panel plots the ratio of the price of the risky asset for n_{fg} relative to n_0, P_{fg}/P_0.

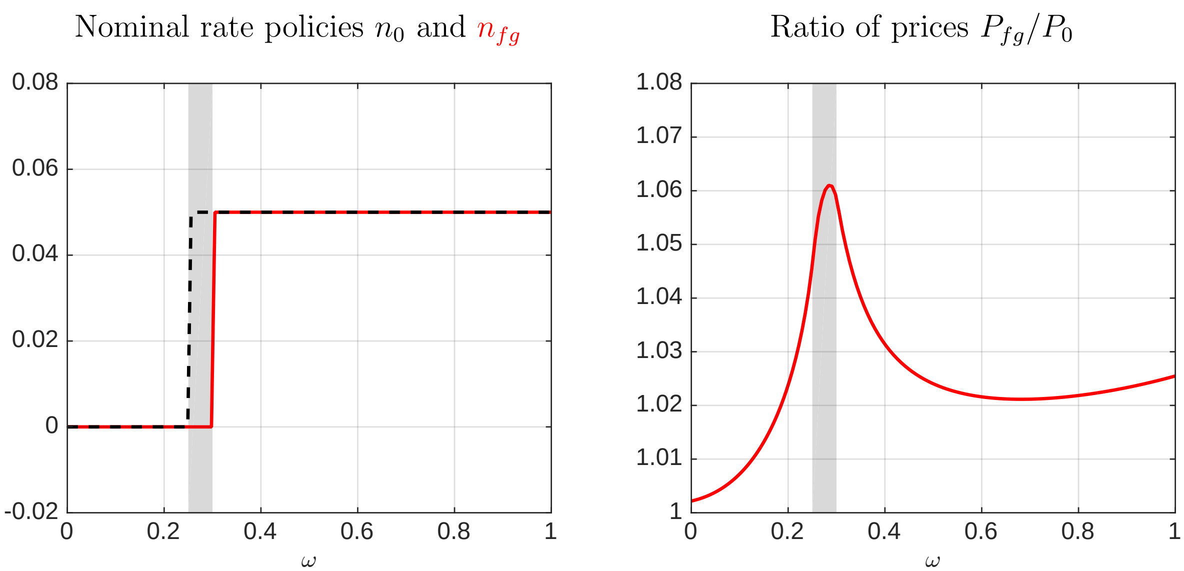

Figure 6: **"Greenspan put" policy and asset prices**

The figure plots the impact of a Greenspan put policy on prices, risk premia, and volatility. The top left panel plots the two nominal rate policies n_0 (dashed black line) and n_{gp} (solid red line). The top right panel plots the price-dividend ratio $P/Y = 1/F$. The bottom left panel plots the risk premium $\mu - r$, and the bottom right panel plots return volatility σ.

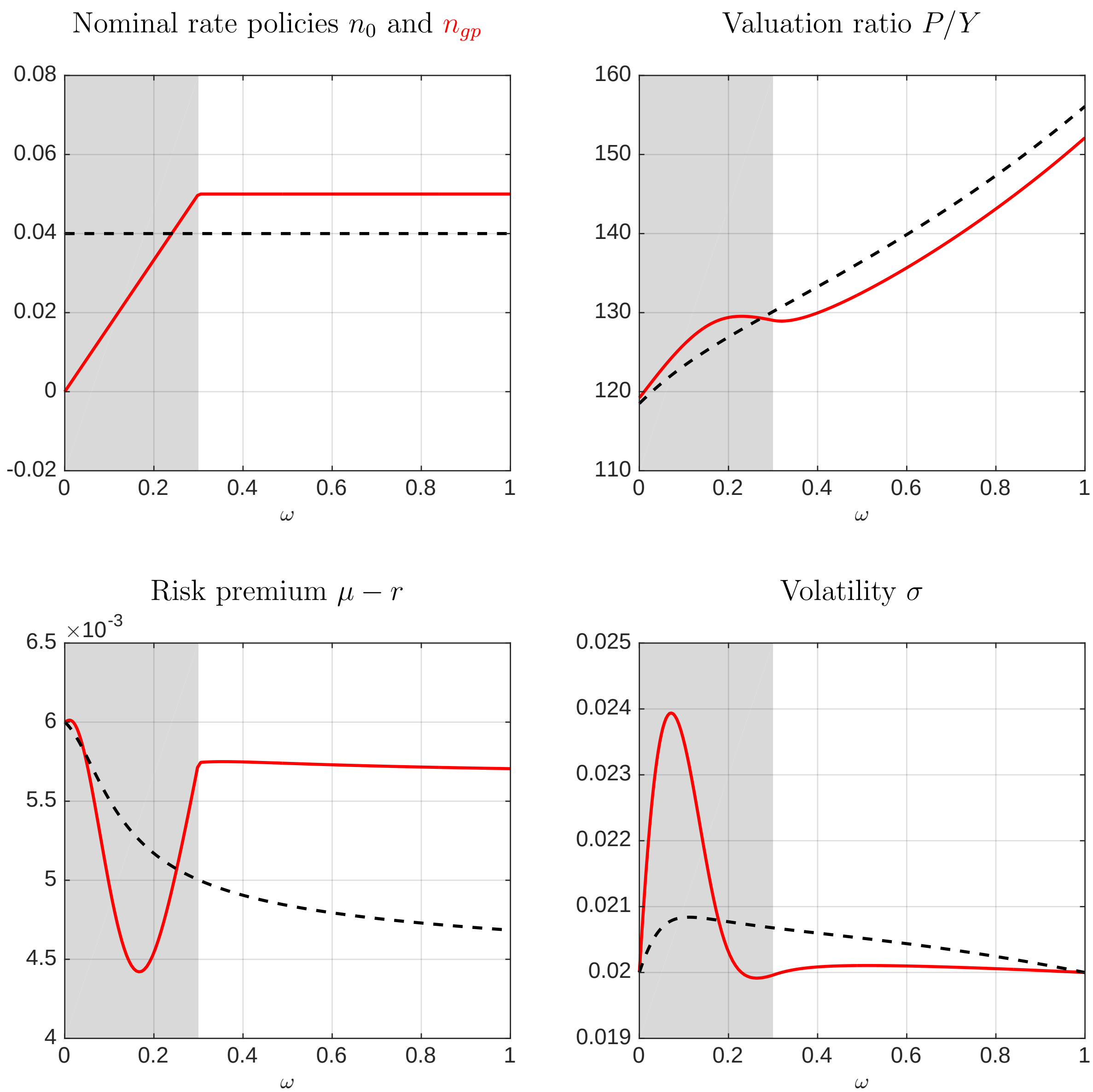

Figure 7: **Nominal rate shocks and financial markets**

Impulse response functions following a 100 bps shock to the nominal rate from the benchmark level $n_0 = 1\%$ and the steady state $\omega_0 = 0.33$. The horizontal axis denotes time. Solid red lines are conditional on the shock, dashed black lines are absent the shock.

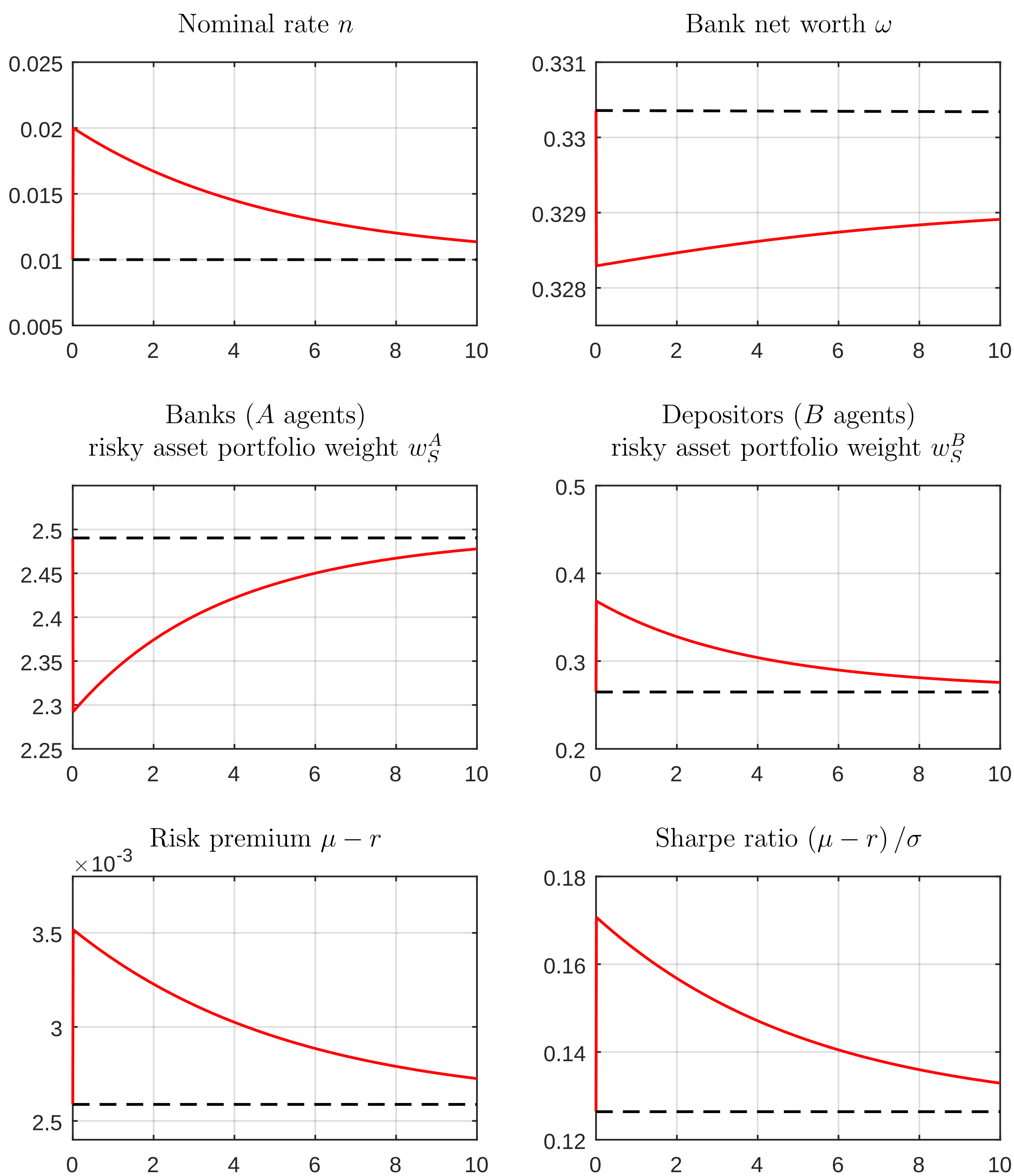

Figure 8: **Nominal rate shocks and economic activity**
Impulse response functions following a 100 bps shock to the nominal rate from the benchmark level $n_0 = 1\%$ and the steady state $\omega_0 = 0.33$. The horizontal axis denotes time. Solid red lines are conditional on the shock, dashed black lines are absent the shock.

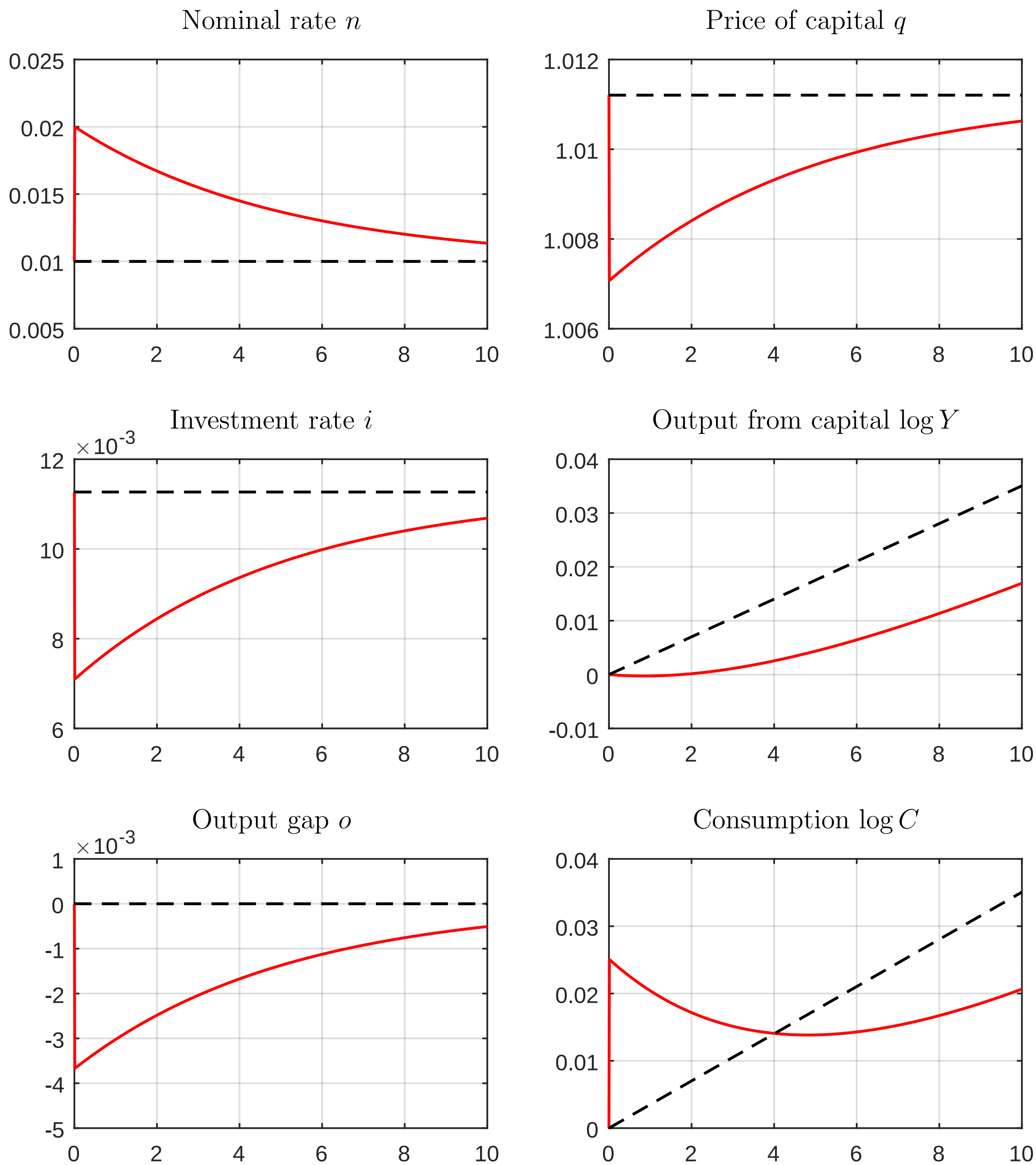

Figure 9: **The term structure of nominal interest rates**

The figure plots the expected path of the nominal rate and the term structure of nominal yields, forward rates, and forward premia before and after a 100bps shock to the nominal rate from the benchmark level $n_0 = 1\%$ and the stochastic steady state $\omega_0 = 0.33$. Nominal bonds are priced at the margin by B agents who are unconstrained. The horizontal axis denotes time for the expected path of the nominal rate and maturity for the term structure plots. Solid red lines condition on the policy shock, dashed black lines do not.