Liquidity and Inefficient Investment

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Abstract

We study the role of fiscal policy in a complete markets model where the only friction is the non-pledgeability of human capital. We show that the competitive equilibrium is constrained inefficient, leading to too little risky investment. We also show that fiscal policy following a large negative shock can increase ex ante welfare. Finally, we show that if the government cannot commit to the promised level of fiscal intervention, the ex post optimal fiscal policy will be too small from an ex ante perspective.

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The Great Recession and the ensuing policy debate on the fiscal stimulus have spurred a renewed interest among economists in the role of government intervention during the business cycle. What is the fundamental economic inefficiency that justifies intervention? Is intervention justifiable only ex post, in the face of an unexpected shock, or are there some ex ante benefits if it is anticipated? If the intervention is anticipated, will the government have an incentive to stick to the ex ante optimal fiscal policy or is there a time inconsistency?

We build a very simple general equilibrium model to answer these questions. We consider an economy subject to an aggregate productivity shock. Suppose that this shock is verifiable and so can be insured against—indeed contingent security markets exist. Assume that if the shock hits, firm profits are low and so is consumer wealth. Then the shock can lead to a fall in demand for consumption goods, and a fall in prices and employment. If there are no imperfections (e.g., sticky prices) there will be no inefficiency: the negative shock lowers welfare but there is nothing that a benevolent government can do to improve matters.

We argue that this conclusion changes if we add just one reasonable friction: consumers cannot pledge their future human capital. Under these conditions (non-human) assets play two roles: they serve as liquidity (because they allow consumers to credibly transfer future income), and they represent part of the consumer's wealth. If the value of pledgeable wealth falls then consumers will cut back on purchases not just because they want to consume less but also because they *have* to consume less.

Anticipating this possibility even risk neutral consumers will want to insure against the aggregate shock by holding an asset whose value is relatively stable, e.g., a riskless asset. We show that such an asset will be held even if it is dominated in expected present value terms by riskier assets. However, we also show that the economy does not get the balance between risky and riskless assets right. Because of liquidity constraints there is an externality: liquidity-constrained consumers over-hoard the riskless asset, driving up prices and hurting other liquidity-constrained consumers who are trying to buy the same goods. We show that the market equilibrium is not even second-best efficient: the government can intervene to obtain a Pareto improvement by restricting investment in riskless assets. In addition, our analysis provides a foundation for certain kinds of fiscal policy that are observed in the real world. We show that a government "gift" of bonds to consumers will increase output and welfare, even if all consumers are ex ante identical and consumers fully anticipate that this gift will be paid for in the future through higher taxes.

Our model, which is a finite horizon one, contains two groups of agents, whom we call doctors and builders. Doctors buy building services from builders and then builders buy doctors' services from doctors, or the other way round. There are large numbers in each group and so markets are competitive.

Each builder requires a doctor at a different date and typically one with different skills from the doctor she is building for, and vice versa for doctors. In other words there is no simultaneous double coincidence of wants (see Jevons (1876) and, for a modern treatment, Kiyotaki and Wright (1989)). Since we assume that future labor income is not pledgeable, this generates a need for means of payment.

Agents are endowed with wheat. Wheat can be invested in various projects. We assume that all asset returns can be pledged but that the returns from some activities are risky. Moreover, risky returns are positively correlated, i.e., there are aggregate productivity shocks. We also suppose that uncertainty about returns is resolved before trading in doctors' and builders' services takes place. Then, a high return realization of risky assets provides a large amount of liquidity for the economy, while a low return realization provides a low amount. Since there are diminishing returns to liquidity—the marginal value of liquidity falls to zero when the gains from trade have been exhausted—this induces the equivalent of risk aversion in agents: the yield on the high return assets is discounted in the good state, and safe assets are favored for liquidity purposes.

We study how competitive firms will invest and issue Arrow securities. In the first-best, the economy operates at full capacity and all wheat is invested in risky projects. In the second-best, however, if the total amount of pledgeable wealth in the bad state is low, the competitive economy will overinvest in the riskless technology and overproduce "money". Investing in liquidity, a doctor who buys building services before he sells doctor services imposes a negative externality on other doctors since his actions increase the price of building services. Moreover, this externality has first order welfare consequences given that doctors are liquidity constrained. Because of this externality too much wheat is invested in the safe asset to create liquidity instead of being invested in socially productive projects.

As a result of this inefficiency, the government can improve on the competitive equilibrium by restricting the amount of investment in the riskless asset.[1] Alternatively, the government can increase the efficiency of the economy by introducing a riskless asset. In our finite horizon economy fiat government money can exist only if the government can tax individuals. We assume that the government can impose sales taxes and that agents can pay these sales taxes with government notes. In our model government notes or government money are equivalent, because both of them must be backed by future taxes. Therefore, since the intervention we consider does not affect the aggregate wealth of consumers, but only the temporal distribution of this wealth, we label it *fiscal policy*.

[1] In Hart and Zingales (2011), we showed that a similar inefficiency and role for government intervention arise in an economy with perfect certainty where there are two investments, one with a high return that is collateralizeable and one with a low return that is not collateralizeable. (As in the current paper human capital cannot be pledged.) One weakness of that formulation is that some assets are *assumed* to be collateralizeable and others not. In a way the current paper can be seen as an attempt to endogenize collateralizeability; an asset with a riskless return serves as better collateral/liquidity than an asset with an uncertain return.

We find that government fiscal policy in bad states not only can increase output more than one-to-one (fiscal multiplier), but also can -- absent any other use of the fiscal lever -- improve ex ante welfare. In fact, our model suggests that it is optimal to undertax in normal times in order to retain the flexibility to use the fiscal lever for this countercyclical policy. We also find that if the government cannot commit to the optimal level of fiscal policy, it will do *too little* of it ex post, i.e. the ex post optimal fiscal policy is different from the ex ante one. This is reminiscent of the famous Kydland and Prescott (1977) result, with two differences: it applies to fiscal policy and it goes in the opposite direction (too little rather than too much).

It should be emphasized that our inefficiency is driven by a single imperfection: the inability of consumers to pledge their future human capital. Markets are complete in other respects. Indeed our results continue to hold even if agents can insure against whether they buy first or sell first. It turns out that an agent who buys first has a lower utility than an agent who sells first but the same marginal utility of wealth. Thus there will be no demand for insurance.

Our work is closely related to Holmstrom and Tirole's recent (2011) book (henceforth HT) and Lorenzoni (2008). Unlike HT and Lorenzoni (2008), our model requires only one friction: the inability of consumers to pledge their future human capital rather than two: the inability of firms to pledge future returns and the inability of consumers to pledge future endowments. Yet, several of the same forces are at play in all these works. As in HT the government's ability to tax gives it a role in improving matters by injecting liquidity, e.g., by creating a riskless asset and taxing consumers later to finance repayments. As in Lorenzoni, the equilibrium without government money creation is second-best inefficient.

Importantly, however, both HT and Lorenzoni focus on firms' liquidity needs, whereas we focus on consumers' liquidity needs. Such a focus seems particularly germane given the growing evidence (Kahle and Stulz (forthcoming) and Mian and Sufi (2012)) that during the Great Recession firms had plenty of liquidity, while consumers were severely liquidity constrained.

This difference in focus changes the sign of the pecuniary externality, leading to opposite results. In Lorenzoni, firms borrow too much, ignoring the fact that in the bad state of the world they have to sell capital to make debt payments, which drives down the price of capital and forces other agents to increase their capital sales (a "fire-sale externality"). In our paper the externality goes the other way: agents acquire too much liquidity (which is like borrowing *too little*), ignoring the fact that their liquidity drives up the price of goods their fellow liquidity-constrained agents are trying to buy (an "inflation" externality). HT do consider an externality similar to ours in Chapter 7. They analyze a situation where ex post some distressed firms will be forced to liquidate and other healthy firms can purchase these firms at discounted prices. They show that ex ante firms will overinvest in safe securities in order to take advantage of the opportunity to buy distressed firms' assets, thereby driving up the prices of these assets and imposing an externality on other buyers.

The different perspectives on the source of the externality lead to opposite policy implications: any ex ante promise to push up prices during a crisis alleviates the inefficiency in case of fire-sale externalities, while it worsens it in the case of our inflation externality. For example, in the context of the last recession, the fire-sale externality perspective would push toward a more aggressive monetary policy, while ours might suggest a renegotiation of underwater mortgages to help consumers who were liquidity constrained. The full difference in policy implications can be seen in Jeanne and Korinek (2013). They develop the ex ante and ex post policy implications of the fire-sale externalities' perspective. As in our case, they find a time inconsistency of fiscal policy. But in their case the fiscal authority would like to commit to intervene *less* than what is ex post optimal to mitigate the overinvestment in *risky* assets; while in our case it would like to commit to intervene *more* than what is ex post optimal to mitigate the overinvestment in *riskless* assets.

These differences in policy implications are a useful reminder of the importance of modeling explicitly the nature of the incompleteness of markets, since the results are very sensitive to the way this is done.

Our model also bears a resemblance to Stein (2012) and Gennaioli et al. (2012). Stein (2012) derives an inefficiency in the provision of money based on the assumption that agents have a discontinuous demand for a riskless claim (money) and that there is a friction in financial markets (patient investors cannot raise additional money). Gennaioli et al. (2012) develop a model in which infinitely risk-averse investors have an appetite for riskless assets and, since these investors ignore unlikely risks, financial intermediaries over-supply these assets. In contrast to these papers we endogenize the demand for riskless assets and any financial frictions and we do not assume any investor irrationality.

Our paper is also related to the huge literature on money. Much of this literature is concerned with the role money plays in general equilibrium (e.g., Hahn (1965)). To create such a role, one needs to dispense with the traditional Walrasian auctioneer and explicitly introduce an exchange process. Ostroy and Starr (1990) provide an excellent survey of attempts in this direction. As far as we can tell, none of these attempts analyze the externality we identify in our paper. The role money plays in our model (i.e., to address the lack of double coincidence of wants) is similar to that in Kyotaki and Wright (1989). Their focus, however, is on what goods can become money and how. Our focus is to what extent private traders can provide the efficient quantity of medium of exchange.

There are parallels between our work and the literature on incomplete markets. In that literature a competitive equilibrium is typically inefficient and a planner operating under the same constraints as the market can do better (see, e.g., Hart (1975) and Geanakoplos and Polemarchakis (1986)). One feature of this literature is that the market structure is taken as given. This raises the question: why can't the private sector create new securities to complete the market? Our work differs in that we endogenize the market structure: markets are complete with respect to verifiable events such as aggregate shocks, but the inability to borrow against human capital creates liquidity problems[2]. Related to this, we focus on whether a market economy overinvests in safe assets, something that, as far as we know, the incomplete markets literature has not considered.

Finally, our paper is linked to a vast and growing literature on the welfare effects of pecuniary externalities in the presence of financial constraints (see, e.g., Kehoe and Levine, (1993), Allen and Gale (2004), Farhi, Golosov, and Tsyvinski (2009), and Korinek (2012)). This literature typically finds that the competitive equilibrium delivers an inefficiently low investment in liquid assets. We find the opposite: a competitive equilibrium delivers an inefficiently high level of liquidity.

2. The Model

Before getting into the details of the model let us provide some explanation of and motivation for its structure. In order to create a role for pledgeability, we consider a situation where two groups trade under competitive conditions but where there is not a double coincidence of wants. This requires two periods of trade: call them periods 2 and 3. Since we are interested in how real investments can alleviate this pledgeability problem, we study the role of firms'securities in providing liquidity. A simple way to introduce firms is to suppose that they transform a numeraire good in period 1 into a numeraire in period 4. Firms' securities as claims on future consumption then provide liquidity in periods 2 and 3.Hence the need for a four-period model.

1 --2------------------------3------------------------------------4

Agents learn whether they will buy or sell first Wheat invested Securities traded	State of world realized	Trade of services	Trade of services	Output from production/ wheat consumed

Figure 1

2 In this respect our paper has parallels with Kiyotaki and Moore (1997), which also endogenizes the market structure, although in a different context.

The two types of agents, whom we call doctors and builders, are assumed to be in equal numbers. As noted, doctors want to consume building services and builders want to consume doctor services. Consumption of these services occurs in periods 2 and 3. Doctors and builders can also consume wheat in period 4 and there is no discounting. Each doctor and builder has an endowment of wheat in period 1 equal to e. We will assume that $e \geq 1$. The timeline is as in Figure 1.

We write agents' utilities as:

Doctors: $U_d = w_d + b_d - \frac{1}{2} l_d^{\,2}$

Builders: $U_b = w_b + d_b - \frac{1}{2} l_b^{\,2}$

where w_i is the quantity of wheat consumed by individual $i = d, b$; b_d is the quantity of building services consumed by a doctor; l_d is the labor supplied by a doctor; d_b is the quantity of doctor services consumed by a builder; and l_b is the labor supplied by a builder. We assume constant returns to scale: one unit of builder labor yields one unit of building services and one unit of doctor labor yields one unit of doctor services.

In words, doctors and builders have a constant marginal utility of wheat, a constant marginal utility of the service provided by the other group of agents, and a quadratic disutility of labor.

Note that the above utility functions have the feature that doctors' and builders' labor supplies will be (somewhat) elastic with respect to price. This will be important when we analyze the role of fiscal policy.

In period 1 each agent learns whether he will first buy or sell. Ex ante both events are equally likely. For convenience, we assume that in the east side of town doctors buy builders' services in period 2, while builders buy doctors' services in period 3. In the west side of town, the order is reversed. (We assume that there is a measure one of doctors and a measure one of builders in each side of town.) In periods 2 and 3 the markets for doctor and building services meet and the doctors and builders trade in the order determined in period 1.

We assume there are many doctors and many builders, and so the prices for both services are determined competitively. It is crucial for our analysis that there is *no simultaneous* double coincidence of wants: builders and doctors are in either the market for buying or the market for selling: they cannot do both at the same time. Hence, even if the builder a doctor buys from wants the doctor services from her customer, she cannot buy them at the same time as she is selling building services.

We have deliberately set up the model to be very symmetric; this helps with the welfare comparisons later. Throughout the paper we will analyze the east part of town, where doctors buy in period 2 and builders in period 3; the reverse case is completely symmetric.

Let's turn now to investment. In period 1 wheat can be invested in two technologies. There is a riskless technology (storage) where one unit of wheat is transformed into one unit of wheat in period 4; and there is a risky technology where one unit of wheat is transformed into R^H >1 units of wheat in period 4 with probability π and $R^L < 1$ units with probability $1-\pi$, where $0 < \pi < 1$ and $\overline{R} = \pi R^H + (1-\pi)R^L > 1$. *The returns of the various risky projects are perfectly correlated.* Thus there are two aggregate states of the world, *R*= R^H or *R*= R^L, which we refer to as high (H) or low (L). Agents learn about the aggregate state of the world between periods 1 and 2. All agents are risk neutral.

We suppose that there is free entry of firms possessing the two technologies described above and that these firms face constant returns to scale.

2.1 A benchmark: the Arrow-Debreu equilibrium

In an Arrow-Debreu economy, the state of the world H or L is verifiable. In addition the penalties for default are infinite. Hence doctors can pledge to pay the builders out of income from supplying doctor services that they will earn in period 3.

It is easy to compute the Arrow-Debreu equilibrium (more precisely, a sequential version). As we will see, the trading economy and the production economy are separable. Consider the trading economy. Once we have reached period 2 all the uncertainty is resolved and so we are in a classic competitive equilibrium model without uncertainty, where there are three goods: doctor services (in period 2), builder services (in period 3), and wheat (in period 4). Imagine that (spot and futures) markets for these three goods open in period 2, and normalize the price of period 4 wheat to be one. Let p_b^H, p_b^L, p_d^H, and p_d^L be the prices of builder and doctor services in the high and low state, respectively, relative to wheat. Focus on one of the states and for the moment leave off the state superscript. Markets cannot clear if either p_b >1 or p_d >1, since demand will be less than supply for building/doctor services, respectively (consumers will prefer wheat). On the other hand, we cannot have both $p_b < 1$ and $p_d < 1$ because then the demand for wheat would be zero, while the supply is positive. Hence, either $p_b < 1$ and $p_d = 1$, or $p_b = 1$, $p_d < 1$, or $p_b = p_d = 1$. It is easy to show that $p_b = p_d$ (we leave this to the reader). We are left with $p_b = p_d = 1$. Utility maximization then implies $b_d = l_b = d_b = l_d = 1$. Each consumer receives a net surplus of ½ from buying one unit of a service and supplying one unit of labor. This is the unique (Walrasian) trading equilibrium. In other words, $p_b^H = p_b^L = p_d^H = p_d^L = 1$.

Now consider the production side. There are markets for two Arrow securities in period 1, one paying a unit of wheat in period 4 in state H and the other paying a unit of wheat in period 4 in state L. Since all parties are risk neutral it is clear that Arrow security prices q^H , q^L will be proportional to probabilities. Also given constant returns to scale neither project can make a profit and at least one must break even. Thus $q^H = \pi / \bar{R}$, $q^L = (1-\pi) / \bar{R}$, where $\bar{R}$ is the expected rate of return on the risky asset, and we normalize the price of period 1 wheat to be one. At these prices only the risky project is employed: all the wheat is invested in that.

Summarizing, we have

Proposition 1: In the unique Arrow-Debreu equilibrium all wheat is invested in the risky project and $p_b^H = p_b^L = p_d^H = p_d^L = 1$, $q^H = \pi / \bar{R}$, $q^L = (1-\pi) / \bar{R}$ *. The utilities of the doctors and builders are* $U_d = e\bar{R} + \frac{1}{2}$, $U_b = e\bar{R} + \frac{1}{2}$*, respectively.*

We see from Proposition 1 that the Arrow-Debreu allocation and prices are independent of the initial endowment *e* and $\bar{R}$ (except for consumption of wheat, which varies one to one with $e\bar{R}$), and each agent's utility is linear in wealth. As we have noted in the Arrow-Debreu equilibrium there is a separation between trade in goods and investment. Given that agents can pledge their human capital, investment income is not needed to finance trade and so investments are chosen entirely on efficiency grounds.

One implication of Proposition 1 is that there is no demand for insurance before an agent learns his type, i.e., whether he will buy first or sell first. Such an insurance market would open before period 1 and pay wheat in period 1 contingent on an agent's type (assumed to be verifiable). That is, a doctor in the east side of town would receive a transfer of wheat from an insurance company, while a builder in the east side of town would give up the same amount of wheat to the insurance company; and vice versa in the west side of town. However, according to Proposition 1, the marginal utilities of wealth of the two types are the same, and hence there is no demand for such insurance in an Arrow-Debreu equilibrium.

2.2 The first-best

Not surprisingly, in light of the first welfare theorem and the absence of a demand for insurance, the Arrow-Debreu equilibrium achieves the first-best. In the first-best the planner maximizes the expected utility of an agent who does not know whether he will buy or sell first (equivalently whether he will be a doctor or a builder) subject to the aggregate feasibility constraints. That is, the planner solves:

$$Max\left\{\pi[w_d^H + b_d^H - \frac{1}{2}(l_d^H)^2 + w_b^H + d_b^H - \frac{1}{2}(l_b^H)^2] + (1-\pi)[w_d^L + b_d^L - \frac{1}{2}(l_d^L)^2 + w_b^L + d_b^L - \frac{1}{2}(l_b^L)^2]\right\}$$

subject to:

$$b_d^H = l_b^H$$

$$d_b^H = l_d^H$$

$$b_d^L = l_b^L$$

$$d_b^L = l_d^L$$

$$w_d^H + w_b^H = y^s + y^r R^H$$

$$w_d^L + w_b^L = y^s + y^r R^L$$

$$y^s + y^r = 2e$$

where w_d^i , w_b^i stands for wheat consumption of doctors and builders in state *i=H,L*, l_d^i , l_b^i, stands for labor services of doctors and builders in state *i*, etc.

The solution is easily seen to be

$$b_d^H = l_b^H = d_b^H = l_d^H = b_d^L = l_b^L = d_b^L = l_d^L = 1$$

$$y^s = 0 \text{ and } y^r = 2e,$$

as in the Arrow-Debreu equilibrium.

2.3 Equilibrium with Non-Pledgeable Labor Income

We now drop the assumption that default penalties are infinite. Instead we assume that doctors who have pledged their period 3 labor income to pay for period 2 goods can just disappear. Knowing this builders will accept only securities as a means of payment in period 2, i.e., doctors will face liquidity constraints. We continue to assume that the state of the world H or L is verifiable, and that markets for the H and L securities exist in period 1. These securities will be supplied by firms investing in projects. Although there are no explicit default penalties, the securities will be collateralized by the project returns in each state and so there will be no default in equilibrium (we are supposing that asset returns cannot be stolen by firms' managers). These securities will be used as a means of exchange in the period 2 and 3 doctor and builder markets.

The timing is: In period 1 markets for wheat (used as input by firms) and Arrow securities open; in period 2 the market for building services opens and doctors use the Arrow securities they purchased in period 1 to buy these services (at this stage, since all the uncertainty is resolved, the Arrow securities are equivalent to riskless bonds); in period 3 the market for doctor services opens and builders use the Arrow securities they purchased in period 1 plus those they have acquired from doctors in period 2 to buy these services; in period 4 Arrow securities pay out and wheat is consumed. Let x_d^H and x_d^L be the quantities of the two Arrow securities bought by doctors and x_b^H and x_b^L the quantities bought by builders. We assume that doctors and builders can disappear in period 4 and so any promise by them to deliver wheat at that time is not credible: hence, x_d^H, x_d^L, x_b^H, $x_b^L \geq 0$. As above, write q^H and q^L as the respective prices of the Arrow securities, relative to period 1 wheat, which is normalized to be one. Let p_b^H, p_b^L, p_d^H, and p_d^L be the prices of builder and doctor services in the high and low state, respectively, relative to period 4 wheat, which is also normalized to be one . Note that price of the H (resp., L) Arrow security in periods 2 and 3 is 1 if H (resp., L) occurs, and 0 otherwise.

Consider a doctor's utility maximization problem. In equilibrium the price of building services in period 2 cannot exceed 1 since otherwise doctors would strictly prefer to use their securities to purchase period 4 wheat rather than building services, and so the building market would not clear. Thus we can assume for the purpose of calculating utility that doctors use all their Arrow securities to buy building services. (By a parallel argument the price of doctor services in period 3 cannot exceed 1 and so for purposes of calculating utility we can assume that builders spend all their Arrow securities on doctor services.) Next consider a doctor's labor supply decision in period 3. Suppose that we have arrived in one of the states, and ignore the superscript on the state. Then a doctor will choose his labor supply l_d to maximize $p_d l_d - \frac{1}{2} l_d^2$, i.e., set $l_d = p_d$. Note that it is too late for the doctor to buy more building services and so his marginal return from work is p_d (he will use the proceeds to buy wheat in period 4). A doctor's labor yields revenue p_d^2, which he redeems for wheat in period 4; in addition he incurs an effort cost of $\frac{1}{2}p_d^2$, and so his net utility is $\frac{1}{2}p_d^2$.

It follows that in period 1 a doctor chooses x_d^H and x_d^L to solve:

$$(*) \qquad \text{Max} \quad \pi\left[\frac{x_d^H}{p_b^H}+\frac{1}{2}\left(p_d^H\right)^2\right]+(1-\pi)\left[\frac{x_d^L}{p_b^L}+\frac{1}{2}\left(p_d^L\right)^2\right]$$

subject to

$$q^H x_d^H + q^L x_d^L \leq e .$$

Note that firm profits are zero in equilibrium given constant returns to scale, and so we do not need to keep track of any dividends received by consumers.

A similar calculation applies to builders. The difference is that a builder in period 2 chooses her labor supply l_b to maximize $\frac{p_b}{p_d}l_b-\frac{1}{2}l_b{}^2$. The reason is that a builder's marginal return from work is $\frac{p_b}{p_d}$, since she will use her income to buy doctor services. Thus a builder's net utility from work is $\frac{1}{2}\left(\frac{p_b}{p_d}\right)^2$. Hence in period 1 a builder chooses x_b^H and x_b^L to solve:

$$(**) \qquad \text{Max} \quad \pi\left[\frac{x_b^H}{p_d^H}+\frac{1}{2}\left(\frac{p_b^H}{p_d^H}\right)^2\right]+(1-\pi)\left[\frac{x_b^L}{p_d^L}+\frac{1}{2}\left(\frac{p_b^L}{p_d^L}\right)^2\right]$$

subject to

$$q^H x_b^H + q^L x_b^L \leq e .$$

Let y^s and y^r be the quantity of period 1 wheat invested respectively in the safe and risky technology. As noted, profit maximization and constant returns to scale imply zero profit: the value of the return stream of each technology cannot exceed the cost of investing in that technology (i.e., 1); and if the inequality is strict the technology will not be used. In other words,

$$(2.1) \qquad q^H + q^L \leq 1 \qquad \text{where } y^s = 0 \text{ if the inequality is strict;}$$

(2.2) $q^H R^H + q^L R^L \le 1$ where $y^r = 0$ if the inequality is strict.

The market clearing conditions in the securities and wheat market in period 1 are given respectively by

(2.3) $x_d^H + x_b^H = y^s + y^r R^H$

(2.4) $x_d^L + x_b^L = y^s + y^r R^L$

(2.5) $y^s + y^r = 2e$

Finally, market clearing conditions in the builder and doctor markets in periods 2 and 3 in each state are:

(2.6) $p_b^H \le 1$. If $p_b^H < 1$, then $\frac{x_d^H}{p_b^H} = \frac{p_b^H}{p_d^H}$. If $p_b^H = 1$, then $x_d^H \ge \frac{1}{p_d^H}$

(2.7) $p_b^L \le 1$. If $p_b^L < 1$, then $\frac{x_d^L}{p_b^L} = \frac{p_b^L}{p_d^L}$. If $p_b^L = 1$, then $x_d^L \ge \frac{1}{p_d^L}$

(2.8) $p_d^H \le 1$. If $p_d^H < 1$, then $\frac{x_b^H + ((p_b^H)^2 / p_d^H)}{p_d^H} = p_d^H$. If $p_d^H = 1$ then $x_b^H + (p_b^H)^2 \ge 1$.

(2.9) $p_d^L \le 1$. If $p_d^L < 1$, then $\frac{x_b^L + ((p_b^L)^2 / p_d^L)}{p_d^L} = p_d^L$. If $p_d^L = 1$ then $x_b^L + (p_b^L)^2 \ge 1$.

(2.6) - (2.9) reflect the fact that, if the price of building (resp., doctor) services is less than one, doctors (resp., builders) want to spend all their income on building (resp., doctor) services. On the other hand, if the price of building or doctor services equals 1, consumers are indifferent between buying the service and wheat and so the market clears as long as liquidity is at least equal to supply.

In summary, the above describes a standard Arrow-Debreu equilibrium with one wrinkle: consumers cannot borrow against future labor income. In what follows we refer to this simply as a "non-pledgeable competitive equilibrium".

The next two lemmas and proposition characterize the non-pledgeable competitive equilibrium and provide a comparison to the Arrow-Debreu equilibrium/ first-best.

Lemma 1: In a non-pledgeable competitive equilibrium, the prices and trading levels of doctor and builder services equal one in the high state ($p_d^H = p_b^H = 1$, and $x_d^H \geq 1$).

Proof: See Appendix.

The (rough) intuition is that, given $R^H > 1$ there is enough liquidity in the high state to support efficient trade at prices of 1.

Lemma 2: A necessary and sufficient condition for a non-pledgeable competitive equilibrium to be first-best optimal is $2eR^L \geq 1$.

Proof: See Appendix.

The (rough) intuition is that if $2eR^L \geq 1$ there is enough liquidity to support efficient trade when all resources are invested in the risky technology, even in the low state.

Proposition 2:

(1) *If $2eR^L \geq 1$, then a non-pledgeable competitive equilibrium delivers the first-best.*

(2) *If $1 > 2eR^L \geq \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$ then a non-pledgeable competitive equilibrium is such that investment is first-best efficient (only the risky technology is used), but trading in doctor and building services is inefficiently low in the low state relative to the first-best.*

(3) *If $2eR^L < \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$ then a non-pledgeable competitive equilibrium is such that investments and trading in labor services are both inefficient relative to the first-best: each technology is operated at a positive scale and trade is inefficiently low in the low state.*

Proof: See Appendix [3]

.

Proposition 2 says that the non-pledgeable competitive equilibrium is in one of three regions. If $2eR^L$ exceeds 1 then we achieve the first-best (which we already know from Lemma 2). If $2eR^L$ is smaller than but close to 1 then there is not enough liquidity in the low state and so trade is inefficient. However, all resources are invested in the risky technology –this follows from the fact that the safe technology is strictly unprofitable at the prices supporting the first-best and so will continue to be unprofitable in a neighborhood of the first-best (see the Appendix). Finally, if $2eR^L$ is much below 1 then

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[3] In each region the competitive equilibrium is unique. We leave the proof to the reader.

the demand for liquidity is sufficiently great relative to the supply that some resources are switched to the safe technology, and so both trade and investment are inefficient.

Note that if $R^L = 0$, the economy will never be at the first-best.

Another way to understand Proposition 2 is as follows. Given that $2eR^L < 1$, what determines the type of distortion is the comparison between the total amount of pledgeable wealth in the bad state and the ratio of the expected capital loss in the bad state ($(1-\pi)(1-R^L)$) to the expected capital gain in the good one ($\pi(R^H - 1)$). If the expected capital loss is small relative to the expected capital gain, then it is still optimal to invest all resources in the risky technology (the first-best outcome) and the inefficiency is limited to trading in doctor and builder services. If the expected capital loss is relatively large, then the optimal allocation will require some investment in the storage technology; this reduces the overall losses in the bad state and also the inefficiency in trading.

At this stage it is worth asking what happens if there is no aggregate uncertainty in the economy (one state of the world). Since the high expected return asset now dominates the storage technology ($\overline{R} > 1$), there is no trade-off between return and liquidity :all investment will be in the high return technology. We have assumed that e≥1 and $\overline{R} > 1$, but let us briefly relax that assumption. There are then two cases. If $2e\overline{R} > 1$, we achieve the first-best (as in Proposition 2; the single state corresponds to the high state). If $2e\overline{R} < 1$,we do not achieve the first-best (the single state corresponds to the low state). However, in this case, there is no externality (see Section 2.4 below), and hence no role for government intervention on the investment side. (In Hart and Zingales (2011) we showed that there will still be an externality even in the perfect certainty case as long as some investments are not fully collateralizeable (see also footnote 1).) Note that even in the perfect certainty case there could be a role for fiscal policy (see Section 3 below). The bottom line is that the certainty case, which can be seen as a finite-horizon version of Woodford (1990), is less rich than the uncertainty case, and that is why we have supposed (aggregate) uncertainty.

2.3 Insurance

A natural question to ask is one we raised with respect to an Arrow-Debreu equilibrium. Since individuals know their type before they trade they could in principle obtain insurance against their type. (Insurance markets would open before period 1 and pay wheat in period 1 contingent on an agent's type, assumed verifiable.) Will they want to do so? As before the answer is no, but the argument is a little more complex since we are in a second-best world. We show in the Appendix that, while the utility of an east side doctor is below that of an east side builder, their marginal utilities of wealth are the same.

Thus there is no demand for insurance.

2.4 Overinvestment in safe assets

We now consider whether a planner operating under the same constraints as the market can do better. We assume that the planner can constrain the production decisions of firms, i.e., choose y^r (or equivalently y^s), but cannot interfere in markets in other ways.

We will focus on the case $2eR^L < \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$. It is easy to see from the proof of Proposition 2 that in a neighborhood of the non-pledgeable equilibrium both technologies are used and $x_d^L > 0, x_b^L = 0.$ Therefore, from (2.4)-(2.5) there is a one-to-one relationship between y^r and x_d^L : decreases in the former correspond to decreases in the latter. In what follows we therefore assume that the planner picks $x_d^L = x^{CP}$ rather than y^r (or equivalently y^s). We will show that the planner can increase surplus by reducing x^{CP} below the non-pledgeable equilibrium level.

Suppose that the planner picks $x_d^L = x^{CP}$ in a neighborhood of the equilibrium. Then the market clearing conditions (2.7) and (2.9) yield

$$p_d^L = \left(x^{CP}\right)^{\frac{1}{2}}$$

$$p_b^L = \left(x^{CP}\right)^{\frac{3}{4}}.$$

Also q_H, q_L will satisfy (2.1)-(2.2) with equality.

The doctors' utility, which is given by $\pi\left[x_d^H + \frac{1}{2}\right] + (1-\pi)\left[\frac{x_d^L}{p_b^L} + \frac{1}{2}\left(p_d^L\right)^2\right]$, becomes $\pi\left[\frac{e-q^L x^{CP}}{q^H} + \frac{1}{2}\right] + (1-\pi)\left[\left(x^{CP}\right)^{\frac{1}{4}} + \frac{1}{2}x^{CP}\right]$. Similarly, the builders' utility, which is given by $\pi\left[x_b^H + \frac{1}{2}\right] + (1-\pi)\left[\frac{1}{2}\left(\frac{p_b^L}{p_d^L}\right)^2\right]$, becomes $\pi\left[\frac{e}{q^H} + \frac{1}{2}\right] + (1-\pi)\left[\frac{1}{2}\left(x^{CP}\right)^{\frac{1}{2}}\right]$.

The planner maximizes $U^d + U^b$. Differentiating the welfare function with respect to x^{CP} yields

(2.10) $$-\pi\frac{q^L}{q^H}+(1-\pi)[\frac{1}{4}\left(x^{CP}\right)^{-\frac{3}{4}}+\frac{1}{2}]+(1-\pi)\frac{1}{4}\left(x^{CP}\right)^{-\frac{1}{2}}$$

We want to prove that (2.10) is negative when we evaluate it at the market equilibrium, $x^{CP}=\left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$. From (2.1) and (2.2) we know that $\frac{1-R^L}{R^H-1}=\frac{q^H}{q^L}$. Hence, we can rewrite (2.10) calculated at $x^{CP}=\left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$ as

$$-\pi\frac{q^L}{q^H}+(1-\pi)[\frac{1}{4}\left(\frac{1-\pi}{\pi}\frac{q^H}{q^L}\right)^{-1}+\frac{1}{2}]+(1-\pi)\frac{1}{4}\left(\frac{1-\pi}{\pi}\frac{q^H}{q^L}\right)^{-\frac{2}{3}}$$

$$=\frac{\pi q^L}{q^H}\left[-1+(x^{CP})^{3/4}((1/2)+(1/4)(x^{CP})^{-3/4}+(1/4)(x^{CP})^{-1/2})\right]$$

<0

since $x^{CP}<1$.

It follows that the planner can increase surplus by reducing x^{CP} below the non-pledgeable equilibrium level, or equivalently by reducing y^s.

Proposition 3: *When* $2eR^L<\left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$ *, the economy overinvests in safe assets.*

In other words, the non-pledgeable competitive equilibrium will be inefficient as long as there is sufficiently high aggregate uncertainty before trading takes place ($2eR^L<\left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$). The intuition is that the non-pledgeability of future labor income creates an additional demand for relatively safe assets. The reason is that transactional needs generate a form of risk aversion even in risk neutral people. When an agent has the opportunity/desire to buy, having a great deal of pledgeable wealth in some states does not compensate her for the risk of having very little pledgeable wealth in other states, because there are diminishing returns to liquidity: in the former states the gains from trade have been exhausted and the marginal value of liquidity is zero, whereas in the latter states the agents are wealth-constrained and the marginal value of liquidity is high. As a result, agents are willing to hold relatively safe assets even if they have a lower yield. However, in doing so they ignore the negative externality they impose on other agents (doctors). This externality arises from the fact that when doctors accumulate more liquidity in the low state this raises the price of building services, which has a first order negative effect on other doctors who are also liquidity constrained. Of course, builders benefit from the increase in price but this effect is second order since builders are on their competitive supply curve.

We have shown that too many resources are invested in manufacturing these relatively safe assets. An example of this is the investment of emerging economies in low-yielding US Treasury securities (see Caballero and Krishnamurthy, 2009). The concern of these economies is precisely that they will not have sufficient collateralizeable wealth in a severe downturn.

Another (domestic) example is the massive investment in assets that were considered at the time as relatively safe (or relatively valuable in the most extreme downturns), such as housing and gold.

2.5 Asset Pricing Implications

In spite of the linear utility of the agents, in case 3 (when $2eR^L < \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$), the price of the high state Arrow security will carry a discount. To see this notice that in case 3 $q^H = \frac{1-R^L}{R^H-R^L}$, while in case 1 (the efficient equilibrium) $q^H = \frac{\pi}{\bar{\bar{R}}}$; it is easy to see that $\frac{1-R^L}{R^H-R^L} < \frac{\pi}{\bar{\bar{R}}}$. Correspondingly, in case 3 the low state Arrow security will carry a premium: in case 3 $q^L = \frac{R^H-1}{R^H-R^L}$, while in case 1 (the efficient equilibrium) $q^L = \frac{(1-\pi)}{\bar{\bar{R}}}$, where it is easy to see that $\frac{R^H-1}{R^H-R^L} > \frac{(1-\pi)}{\bar{\bar{R}}}$.

This result is reminiscent of Barro (2006), who rationalizes the high equity premium and low risk free rate on the basis of the historical occurrence of rare disasters. Our low state with $2eR^L < \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$ can be thought of as one of those disasters. The main difference is that Barro rationalizes the equity premium and low risk free rate in the context of a representative agent model with risk-averse investors, while we do so with heterogeneous agents who are risk neutral.

3. Fiscal Policy

So far we have ignored the role of the government in providing liquidity. We will now relax this assumption. Following Holmstrom and Tirole (1998, 2011) and Woodford (1990), we assume the government can exploit a power it has, which the private sector does not have: the power to tax. In particular, the government can issue notes to consumers, and these notes will be valuable because they are backed by future tax receipts. In our finite horizon model notes or money are equivalent, because both of them must be backed by future taxes. Therefore, since the intervention we consider does not affect the aggregate wealth of consumers, but only the temporal distribution of this wealth, we label it *fiscal policy*.

Holmstrom and Tirole (1998) justify the assumption that it is easier for the government to collect taxes, than for creditors to collect debts, from consumers on the grounds that the government can audit incomes or impose jail penalties. While realistic this assumption can be criticized on the grounds that in principle at least jail could be used as a penalty for the non-payment of private debts—after all debtors' prisons have existed in the past. We therefore adopt a different rationale. We suppose that the government can impose sales taxes on certain productive facilities that consumers use and which can be easily monitored. Private lenders cannot duplicate such an arrangement since they do not have the power to require (all) facilities to participate.

To allow for sales taxes we assume that in period 4 our agents consume flour as well as wheat: one unit of flour yields one unit of utility. There is a milling technology for turning wheat into flour: each agent can obtain λ units of flour at the cost of $\frac{1}{2}c\lambda^2$ units of wheat, where $\lambda \geq 0$ is the agent's choice variable. This activity occurs at facilities (mills) that can easily be monitored by the government, and so the government can impose a per unit flour tax t that cannot be avoided.

An agent's utility is now:

Doctors: $U_d = w_d + b_d - \frac{1}{2}{l_d}^2 + (1-t)\lambda_d - \frac{1}{2}c{\lambda_d}^2$

Builders: $U_b = w_b + d_b - \frac{1}{2}{l_b}^2 + (1-t)\lambda_b - \frac{1}{2}c{\lambda_b}^2$

where t is the tax rate on flour.

We assume that in period 4 each agent has a large endowment of wheat (in addition to any labor income and dividends from investment), so that she is not at a corner solution, and hence λ_d, λ_b satisfy the first order condition

$$\lambda_d = \lambda_b = \frac{1-t}{c}.$$

This yields

$$U_d = w_d + b_d - \frac{1}{2} l_d^{\,2} + \frac{1}{2c}(1-t)^2 ,$$

$$U_b = w_b + d_b - \frac{1}{2} l_b^{\,2} + \frac{1}{2c}(1-t)^2 .$$

The total amount raised by taxes on doctors and builders is

$$T = \frac{2t(1-t)}{c} . \tag{3.1}$$

Since in the high state there is ample liquidity, it is natural to focus on the case where the government issues notes only in the low state. We will suppose that the government can target those who need the liquidity most: the doctors. (Our analysis so far is consistent with the assumption that it is verifiable in period 2 whether an agent has to buy or sell first.)[4]

In summary, the government's fiscal policy consists of issuing *m* notes to each doctor in period 2 if and only if the state L occurs. Each note promises one unit of wheat in period 4.

Given that government notes must be backed by taxes, we have

$$m = T = \frac{2t(1-t)}{c} . \tag{3.2}$$

Note that (3.2) implies that $T=0$ when $t=0$ and T reaches a maximum at $t = \frac{1}{2}$. Thus, it is never optimal to set $t > \frac{1}{2}$ since the deadweight loss increases in t

From a period 2 perspective this very simple model exhibits some Keynesian features when $2eR^L < \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H - 1} \right)^{\frac{4}{3}}$. If, when the state is low, the government intervenes with an (unexpected) hand-out *m* in period 2, it will have the effect of boosting the level of output by more than *m* (fiscal multiplier). To see this, assume that x_d^L and x_b^L are fixed at their competitive equilibrium levels, which are less than 1. Consider a small government hand-out of m to the doctors. Then, after the hand-out the new equilibrium becomes:

$$\frac{x_d^L + m}{p_b^L} = \frac{p_b^L}{p_d^L}$$

$$\frac{x_d^L + m}{p_d^L} = p_d^L ,$$

[4] The case where the government cannot target the constrained agents is not qualitatively different, but a bit more cumbersome.

which implies

$p_b^L = (x_d^L + m)^{\frac{3}{4}}$ and $p_d^L = (x_d^L + m)^{\frac{1}{2}}$. Since $l_d^L = p_d^L$ and $l_b^L = \frac{p_b^L}{p_d^L}$, the fiscal policy increases (the nominal value of) output (which we measure as $p_d^L l_d^L + p_b^L l_b^L = (p_d^L)^2 + \frac{(p_b^L)^2}{p_d^L}$) from $2x_d^L$ to $2(x_d^L + m)$.

Thus, the fiscal multiplier is 2. The intuition behind the multiplier is very Keynesian: the fiscal stimulus enables the doctors to buy more building services. This extra demand increases production by builders and therefore builders can afford to buy more from doctors, raising their production. In turn, this additional production enables doctors to pay for what they bought from the builders and the additional taxes.

Not only does a fiscal policy following a big negative shock increase output more than one-to-one, but it also increases ex ante welfare. To see this it is sufficient to notice that ex post welfare in state L is given by

(3.3) $$W^L = \left[\frac{x_d^L + m}{p_b^L} + \frac{1}{2}\left(p_d^L\right)^2 + \frac{1}{2c}(1-t)^2 + \frac{x_b^L}{p_d^L} + \frac{1}{2}\left(\frac{p_b^L}{p_d^L}\right)^2 + \frac{1}{2c}(1-t)^2 \right],$$

and

(3.4) $$\frac{\partial W^L}{\partial m} = [\frac{1}{4}(x_d^L + m)^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4}(x_d^L + m)^{-\frac{1}{2}} - \frac{(1-t)}{(1-2t)}] > 0 \text{ for } m \text{ close to } 0.$$

For future reference note that the optimal fiscal intervention in the non-anticipated case is characterized by

(3.5) $$[\frac{1}{4}(x_d^L + m)^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4}(x_d^L + m)^{-\frac{1}{2}} - \frac{(1-t)}{(1-2t)}] = 0.$$

Since t is increasing in m it is easy to see that (3.5) has a unique solution.

A more interesting question is what happens when the intervention is fully anticipated. We will consider two cases: one where the government can commit to *m* ex ante and the other where the government cannot commit. Also in contrast to Section 2.4 we will suppose that the government cannot constrain the production decision of firms: its only policy tool is *m*.

3.1. The case of commitment

We will assume that agents have rational expectations about government actions. Suppose that agents anticipate that *m* will be injected in the low state. Then, the equilibrium of Section 2.2 changes as follows:

In period 1 a doctor chooses x_d^H and x_d^L to solve:

$$(*)\qquad \text{Max}\quad \pi\left[\frac{x_d^H}{p_b^H}+\frac{1}{2}\left(p_d^H\right)^2+\frac{1}{2c}\right]+(1-\pi)\left[\frac{x_d^L+m}{p_b^L}+\frac{1}{2}\left(p_d^L\right)^2+\frac{1}{2c}(1-t)^2\right]$$

subject to

$$q^H x_d^H + q^L x_d^L \le e\,.$$

where (*) reflects the fact that the flour tax is zero in state H and *t* in state L and the doctors receive *m* in state L.

Similarly, a builder chooses x_b^H and x_b^L to solve:

$$(**)\qquad \text{Max}\quad \pi\left[\frac{x_b^H}{p_d^H}+\frac{1}{2}\left(\frac{p_b^H}{p_d^H}\right)^2+\frac{1}{2c}\right]+(1-\pi)\left[\frac{x_b^L}{p_d^L}+\frac{1}{2}\left(\frac{p_b^L}{p_d^L}\right)^2+\frac{1}{2c}(1-t)^2\right]$$

subject to

$$q^H x_b^H + q^L x_b^L \le e\,.$$

The other equilibrium conditions (2.1)-(2.6) and (2.8)-(2.9) stay the same, while (2.7) becomes

$$(3.6)\qquad p_b^L \le 1.\ \text{If}\ p_b^L < 1,\ \text{then}\ \frac{x_d^L+m}{p_b^L}=\frac{p_b^L}{p_d^L}.\ \ \text{If}\ p_b^L = 1,\ \text{then}\ x_d^L + m \ge \frac{1}{p_d^L}$$

The government chooses in period 0 to maximize the expected utility of an agent who does not know whether he will buy or sell first (equivalently, whether he will be a doctor or a builder). That is, the government chooses *m* to maximize the sum of doctor and builder utilities:

$$(3.7)\qquad W=\pi\left[\frac{x_d^H}{p_b^H}+\frac{1}{2}\left(p_d^H\right)^2+\frac{1}{2c}+\frac{x_b^H}{p_d^H}+\frac{1}{2}\left(\frac{p_b^H}{p_d^H}\right)^2+\frac{1}{2c}\right]+$$

$$(1-\pi)\left[\frac{x_d^L+m}{p_b^L}+\frac{1}{2}\left(p_d^L\right)^2+\frac{1}{2c}(1-t)^2+\frac{x_b^L}{p_d^L}+\frac{1}{2}\left(\frac{p_b^L}{p_d^L}\right)^2+\frac{1}{2c}(1-t)^2\right]$$

where for each *m* the *x*'s and the *p*'s are given by the market equilibrium corresponding to that *m*.

The interesting case is when in the absence of fiscal policy investment and trading in labor services are both inefficient, i.e. $2eR^L < \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$ (see Proposition 2). In this case we have

Proposition 4: If $2eR^L < \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$, a positive injection of notes ($m > 0$) in the low state is welfare improving.

Proof:

We know from the proof of Proposition 2 that the competitive equilibrium in the case *m=0* has the following features :

(3.8) Doctors hold both securities, and so

$$\frac{\pi}{q^H} = \frac{1-\pi}{p_b^L q^L},$$

(3.9) Both technologies are used and so $q^L = \frac{R^H-1}{R^H-R^L}$ and $q^H = \frac{1-R^L}{R^H-R^L}$,

(3.10) $$x_b^L = 0 \ , \quad p_b^L = \frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1} < 1 \ .$$

Furthermore, it is easy to adapt the proof of Proposition 2 to show that (3.8)-(3.10) will hold in a competitive equilibrium when the government injects *m*, for *m* close to zero. The market clearing conditions (2.9), (3.6) can be written as

(3.11) $$\frac{x_d^L+m}{p_b^L} = \frac{p_b^L}{p_d^L},$$

(3.12) $$\frac{x_d^L + m}{p_d^L} = p_d^L ,$$

which imply

(3.13) $$p_d^L = (p_b^L)^{\frac{2}{3}},\ x_d^L + m = (p_b^L)^{\frac{4}{3}} .$$

According to (3.10), the government injection of *m* does not affect p_b^L. Hence, from (3.13), p_d^L stays the same and so does $x_d^L + m$. We may conclude that there is a 100% crowding out of x_d^L by *m*.

The market clearing conditions, (2.3)-(2.5), imply

(3.14) $$y^s = \frac{x_d^L - 2eR^L}{1 - R^L}$$

(3.15) $$y^r = \frac{2e - x_d^L}{R^L - 1}$$

Hence, a fall in x_d^L leads to a decrease in y^s, an increase in y^r, and an increase in $x_d^H + x_b^H$.

In summary, we have

(3.16) $$\frac{dx_d^L}{dm} = -1,\ \frac{dy^s}{dm} = -\frac{1}{1 - R^L},\ \frac{dy^r}{dm} = \frac{1}{1 - R^L} .$$

Also from (3.2)

(3.17) $$\frac{dt}{dm} = \frac{c}{2(1-2t)} .$$

To ascertain the effect of *m* on welfare we differentiate (3.7) with respect to *m* to obtain:

(3.18) $$\frac{dW}{dm} = \pi[\frac{dy^s}{dm} + \frac{dy^r}{dm}R^H] + (1-\pi)[-\frac{2}{c}(1-t)\frac{dt}{dm}] = \pi[\frac{R^H - 1}{1 - R^L}] - (1-\pi)\frac{(1-t)}{(1-2t)}$$

Hence, the optimal choice of *m* is characterized by the first order condition:

(3.19) $$\frac{\pi}{(1-\pi)}[\frac{R^H - 1}{1 - R^L}] \le \frac{(1-t)}{(1-2t)} \text{ with equality if } m > 0 .$$

Note that the LHS of (3.19) is strictly bigger than 1 given that the expected return of the risky asset strictly exceeds one. Thus, (3.19) cannot be satisfied at *t=0*. We may conclude that a government hand-out ($m > 0$) in the low state is welfare improving.

QED

We see that, when the government intervention is expected, the inefficient overinvestment in safe assets is reduced. Yet, the level of output in periods 2 and 3 is still inefficient. As (3.16) makes clear, government liquidity completely crowds out private liquidity. As a result, the level of trade remains the same as in the original equilibrium without government intervention. Nevertheless, when the government does intervene in period 2, the multiplier is bigger than 1 as per the analysis above.

Given this tension, what is the optimal level of *m*? Assume that *c* is small (the deadweight cost of taxation is large) and thus it is never desirable for the government to move the economy too far from the non-intervention equilibrium. We can then be confident that we will remain in a neighborhood of $m=0$ and so (3.8)-(3.10) continue to hold. The first order condition for the optimal *m* then follows from (3.19):

(3.20)
$$\frac{\pi}{(1-\pi)}[\frac{R^H-1}{1-R^L}]=\frac{(1-t)}{(1-2t)}$$

Since the RHS is strictly increasing in *t* and converges to ∞ as $t\rightarrow\frac{1}{2}$ from below, (3.20) has a unique solution and the optimal *m* can be deduced from (3.20).

In the above we have ignored the possible existence of other government expenses (e.g., to finance valuable public goods). If they exist, then distortions of extra taxation are first order, not second order. In such a case it is not obvious that we want to intervene with a fiscal stimulus. In fact, if the government anticipates the need for such a stimulus in the future it might want to restrain the provision of public goods (and taxation) beyond what it optimal in isolation to retain the fiscal flexibility to intervene in case of a disaster.

In the above we have also considered optimal fiscal policy when fiscal policy is the only instrument at the government's disposal. A natural question to ask is what happens if the government can choose fiscal policy *and* control the investment allocation. It is not difficult to show that fiscal policy is still useful : *m* >0 at the optimum. We leave the details to the reader.

3.2. The case of non-commitment

Suppose that *m* is characterized by (3.20) but now the government can change *m* ex post if state L occurs. Will it choose to do so? We assume that the government continues to be benevolent: it maximizes the sum of doctor and builder utilities in the low state. The problem in term of commitment is that x_d^L , x_b^L and production decisions are sunk.

Given that x_d^L and $x_b^L=0$ are fixed, market prices for doctor and building services will be given by (3.11) –(3.12), where *m* now varies. Total welfare in the low state (see (3.3)) can be written as

(3.21) $$W^L = \left[(x_d^L + m)^{\frac{1}{4}} + \frac{1}{2}\left(x_d^L + m\right) + \frac{1}{2}\left(x_d^L + m\right)^{\frac{1}{2}} + \frac{1}{c}(1-t)^2 \right].$$

Since x_d^L is fixed,

(3.22) $$\frac{\partial W^L}{\partial m} = [\frac{1}{4}(x_d^L + m)^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4}(x_d^L + m)^{-\frac{1}{2}} - \frac{(1-t)}{(1-2t)}].$$

Now apply (3.20) and use (3.11)-(3.12) to write $x_d^L + m = (p_b^L)^{\frac{4}{3}}$. Then,

(3.23) $$\frac{\partial W^L}{\partial m} = [\frac{1}{4}(p_b^L)^{-1} + \frac{1}{2} + \frac{1}{4}(p_b^L)^{\frac{-2}{3}} - (p_b^L)^{-1}] < 0$$

since $p_b^L < 1$.

We see that, when the government considers its decision as of period 2, it has an incentive to renege on the previously announced level of fiscal intervention. In other words,

Proposition 5: The government fiscal policy is time inconsistent.

Ex post the government will want to give fewer hand-outs than it said it would. The reason is that the promise to give hand-outs in the low state helps address two problems: the inefficient investment in period 0 and the inefficiently low level of trade in periods 2 and 3. If the government can renege on its promise in period 2, however, it will find that at that time its actions affect only one inefficiency: the low level of trade in periods 2 and 3. Since the government finds it less beneficial to tax people to deal with one inefficiency rather than two, it will deviate in the direction of intervening less than promised.

Note that the optimal time-consistent fiscal policy is characterized by

(3.24) $$[\frac{1}{4}(x_d^L + m)^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4}(x_d^L + m)^{-\frac{1}{2}} - \frac{(1-t)}{(1-2t)}] = 0,$$

where $x_d^L + m = (p_b^L)^{\frac{4}{3}} = \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H - 1} \right)^{\frac{4}{3}}$.

3.3 Comparison of fiscal policies

How do the fiscal policies for the three cases—unanticipated, commitment, and non-commitment—differ? We already know that the commitment level of m is higher than the non-commitment level. If we compare the non-commitment level with the unanticipated level we see that the first order conditions (3.5) and (3.24) are the same. However, since there is crowding out when m is anticipated the level of x_d^L is lower in (3.24) than in (3.5). It follows that the value of m satisfying (3.24) will be higher than that satisfying (3.5): the non-commitment level is higher than the unanticipated level. The intuition is that when fiscal policy is not anticipated the desirable crowding out effects do not occur and so it is less attractive.

Thus the ranking of fiscal policies from high to low is: commitment, non-commitment, unanticipated.

4. An Extension

So far, we have considered borrowers who can breach any promise to pay future labor income by "disappearing". In practice some specialist agents may be able to keep track of borrowers and force them to repay their debts. Specifically, suppose that all payments for building and doctor services take place through check transfers and that a bank is able to seize these before they are cashed for consumption. In this way labor income becomes contractible. However, the bank cannot force anyone to work. That is, all the bank can do is to ensure that someone who defaults has zero consumption. As a result, uncollateralized lending against future labor income is possible, but there is a repayment constraint. Each worker can borrow up to the point at which ex post he is indifferent between exerting effort and repaying the loan and doing nothing and defaulting.

We analyze this case in a simplified setting in Hart and Zingales (2011), and show that our results generalize. In particular, lending does not resolve the tension between private and social objectives. However, lending does improve welfare since it increases the volume of trade without sacrificing the higher return of the risky investment. Interestingly, lending is not a perfect substitute for the riskless asset. The reason is that with no private liquidity in the system, the amount borrowed by doctors equals the purchasing power in the hands of builders, which in turn equals the revenue received by doctors for their services. But if the revenue equals the debt, it is not in the interest of the doctors to work, given that they have to exert costly effort. Hence, the doctors will default. To have a functioning lending market, we need a minimum amount of private liquidity.

5. Conclusions

We have built a simple framework to analyze the role of fiscal policy in attenuating the impact of aggregate shocks on private investment choices and aggregate output. We show that the mere lack of pledgeability of human capital, even in the presence of complete markets for securities, makes the competitive equilibrium constrained inefficient. The market will invest too much in producing safe securities and will dedicate too few resources towards risky investments.

Our work can be distinguished from that of Holmstrom and Tirole (2011) and Lorenzoni (2008). Their analyses are based on the idea that the market underprovides liquidity to firms, whereas in our treatment consumer liquidity needs are the driver. To compare the importance of the two, we looked at the Survey of Small Businesses Finances and Survey of Consumer Finances. The 2004 Survey of Consumer Finances finds that 37% of families are financially constrained, where constrained is defined as a family that applied for credit and has been rejected or has been discouraged from applying by the fear of being rejected. By contrast, the 2003 Survey of Small Businesses Finances finds that only 15% of small firms were constrained, using the same definition. Since small firms are more likely to be constrained than big firms, this evidence seems to suggest that financial constraints are more likely to be a problem for consumers than for firms.

In our simple model a fiscal policy following a big negative shock can increase not only ex post output more than one-to-one (fiscal multiplier), but also ex ante welfare. We have supposed that the government is able to target directly consumers who are in need of liquidity. If we were to drop this assumption, an interesting set of problems would arise. Would it be cheaper for the government to bail out financial intermediaries rather than to hand out money to consumers randomly? If so, how would this benefit trade off against the potential moral hazard problem financial intermediaries face when they expect to be bailed out in major downturns? We analyze these and other issues in a separate paper (Hart and Zingales, 2013).

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Appendix

Proof of Lemma 1: Suppose $p_d^H < 1$ and $p_b^H < 1$. Then, by (2.8) and (2.6), $x_b^H + x_d^H = (p_d^H)^2 < 1$, which contradicts (2.3).

Now suppose $p_d^H < 1$ and $p_b^H = 1$. Then, by (2.8), $x_b^H = (p_d^H)^2 - \frac{1}{p_d^H} < 0$, which is impossible.

Hence $p_d^H = 1$.

To prove that $p_b^H = 1$, assume the contrary: $p_b^H < 1$. We first show that $x_d^H \geq x_d^L$. Suppose not: $x_d^H < x_d^L$. Then $x_d^L > 0$. From the first order conditions for (*),

(A1) $$\frac{\pi}{p_b^H q^H} \leq \frac{1-\pi}{p_b^L q^L}$$

That is, the utility rate of return on the low state Arrow security for doctors must be at least as high as that on the high state Arrow security. We also know that there is more output in the high state, so, if $x_d^H < x_d^L$, builders must be buying the high state security, which means that it must give them an attractive return, or , from their first order condition,

(A2) $$\frac{\pi}{q^H} \geq \frac{1-\pi}{p_d^L q^L},$$

where we are using the fact that $p_d^H = 1$.

Putting (A1)-(A2) together yields

(A3) $$\frac{p_b^L}{p_b^H} \leq p_d^L.$$

If $p_b^L = 1$, (A3) implies $p_b^H = 1$, which we have supposed not to be the case. Hence $p_b^L < 1$. Then we have, from (2.6) and (2.7), $\left(p_b^H\right)^2 = x_d^H$ and $\left(p_b^L\right)^2 = x_d^L p_d^L$. Therefore (A3) becomes

$$\frac{\left(x_d^L\right)^{\frac{1}{2}}}{\left(x_d^H\right)^{\frac{1}{2}}} \leq \left(p_d^L\right)^{\frac{1}{2}} \leq 1$$

or $x_d^H \geq x_d^L$, which is a contradiction.

Hence, $x_d^H \geq x_d^L$. Since a doctor's utility is increasing in x_d^L and x_d^H, a doctor's budget constraint will hold with equality. Thus $q^H x_d^H + q^L x_d^L = e$, which implies $(q^H + q^L)x_d^H \geq e$. Hence, by (2.1), $x_d^H \geq e > 1$, implying $p_b^H = 1$ by (2.6).

Q.E.D.

Proof of Lemma 2:

To achieve the first-best the supply of building and doctor services must be 1 in each state. Since the supply of doctor services is given by p_d^H, p_d^L in (2.8), (2.9), it follows that $p_d^H = p_d^L = 1$. The supply of building services is given by $\frac{p_b^H}{p_d^H}, \frac{p_b^L}{p_d^L}$ in (2.6)-(2.7), and, substituting $p_d^H = p_d^L = 1$, we obtain $p_b^H = p_b^L = 1$. Hence, again from (2.6)-(2.7), $x_d^H \geq 1$ and $x_d^L \geq 1$.

In the first-best all wheat is invested in the high yield project: $y^s = 0$ and $y^r = 2e.$ Therefore, from (2.4), $2eR^L = x_d^L + x_b^L \geq x_d^L \geq 1$. Hence, $2eR^L \geq 1$ is a necessary condition.

To prove sufficiency consider a candidate equilibrium where the prices of doctor and builder services equal 1 in both states, $q^H = \frac{\pi}{\bar{R}}$, $q^L = \frac{(1-\pi)}{\bar{R}}$, all wheat is invested in the high yield project , and the doctors buy at least one unit of each Arrow security. Since $q^H + q^L < 1 \leq e$, they can afford to do so. Doctors and builders satisfy their first order conditions and firms maximize profit. Hence this is indeed a competitive equilibrium.

Q.E.D.

Proof of Proposition 2:

Lemma 2 shows that we achieve the first-best if $2eR^L \geq 1$. Consider the case $2eR^L < 1$. We know from Lemma 1 that $p_d^H = p_b^H = 1$. We show first that doctors will hold both securities. Given p_b^H =1, (2.6) implies x_d^H >0. Suppose $x_d^L = 0$. This is inconsistent with $p_b^L = 1$ in (2.7). But if $p_b^L < 1$, then, from

(2.7), $x_d^L = 0$ implies $p_b^L = 0$. This in turn implies that the marginal return on the low state Arrow security for a doctor is infinite, which means that the first order condition in (*) cannot hold. Therefore, $x_d^L > 0$.

Since doctors hold both securities, we have

(A4) $$\frac{\pi}{q^H} = \frac{1-\pi}{p_b^L q^L}$$

Let's assume first that both technologies are used. Then (2.1) and (2.2) hold as an equality and $q^L = \frac{R^H - 1}{R^H - R^L}$ and $q^H = \frac{1 - R^L}{R^H - R^L}$. Therefore

(A5) $$p_b^L = \frac{1-\pi}{\pi}\frac{1-R^L}{R^H - 1} < 1$$

since $\bar{R} > 1$.

It is easy to see that $p_b^L < p_d^L$. This is clear from (A5) if $p_d^L = 1$. Suppose $p_d^L < 1$. Then (2.9) implies $\left(p_d^L\right)^2 = x_b^L + \frac{(p_b^L)^2}{p_d^L} \geq \frac{\left(p_b^L\right)^2}{p_d^L}$. Hence, $p_d^L \geq (p_b^L)^{\frac{2}{3}} > p_b^L$ since $p_b^L < 1$. This proves $p_b^L < p_d^L$. It follows that the rate of return on the low security is strictly less than that on the high security for builders. So builders will not hold the low security (from the first order condition for (**)): $x_b^L = 0$.

From (2.7),

(A6) $$p_d^L x_d^L = \left(p_b^L\right)^2,$$

from which it follows, since $p_b^L < 1$ and $p_b^L < p_d^L$, that $x_d^L < 1$. But then (2.9) in combination with $x_b^L = 0$ implies $p_d^L < 1$. Hence, again from (2.9),

(A7) $$x_d^L = \left(p_d^L\right)^2.$$

Combining (A6) and (A7) we have

(A8) $$x_d^L = \left(p_b^L\right)^{\frac{4}{3}}.$$

Hence

(A9) $$p_d^L = (\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1})^{2/3}, x_d^L = (\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1})^{4/3}.$$

If the solution $x_d^L = \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}} > 2eR^L$, then this candidate equilibrium is feasible. Note that both technologies are used: since $x_d^L > 2eR^L$ the riskless technology must be used and since, by Lemma 1 and (A7), $x_d^H \geq 1 > x_d^L = x_d^L + x_b^L$ the risky technology must also be used. Also trade of doctor and builder services is inefficient since p_b^L, p_d^L are both less than 1.

If $x_d^L = \left(\frac{1-\pi}{\pi}\frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}} \leq 2eR^L$, then we solve instead for an equilibrium with $x_d^L = 2eR^L$, $x_b^L = 0$. In this case, there will be no investment in the storage technology (thus $y^s = 0$) and the market clearing condition for securities, (2.3)-(2.4), simplifies to

$$x_d^H + x_b^H = 2eR^H,$$
$$x_d^L + x_b^L = 2eR^L.$$

(2.7) and (2.9) become

$$p_d^L = \left(2eR^L\right)^{\frac{1}{2}}$$

$$p_b^L = \left(2eR^L\right)^{\frac{3}{4}},$$

which are below one since $2eR^L < 1$. Doctors hold both Arrow securities since $p_b^H, p_b^L > 0$ and so we must have

$$\frac{\pi}{q^H} = \frac{1-\pi}{p_b^L q^L} = \frac{1-\pi}{\left(2eR^L\right)^{\frac{3}{4}} q^L}.$$

This, together with (2.2) with equality, determines q^H and q^L. Thus, in this equilibrium investment is efficient but the level of trading is not. QED

Proof that Insurance Markets have no role

We demonstrate that, even in the second-best, there is no role for markets that open before period 1 and pay wheat in period 1 contingent on an agent's type (assumed verifiable). For simplicity, let us focus on

Case 3 of Proposition 2, where there is inefficiency in both investment and trade. (The result applies to the other cases of Proposition 2 as well.)

Insurance redistributes endowments between those who sell before they buy and those who buy before they sell; equivalently, since we focus on the east side of town, between builders and doctors. That is, insurance entails a doctor receiving a transfer θ , $-e \le \theta \le e$, from a builder. (We expect θ>0 but in principle we could have θ<0.) Note that much of the proof of Lemma 1 applies for any θ: in particular, $p_d^H = 1$ and $x_d^H \ge x_d^L$. It is easy to rule out the case where $\theta = -e$, i.e., doctors have no wealth ex post. To see this note that by the doctor budget constraint we would have $x_d^H = x_d^L = 0$; hence $x_b^H, x_b^L > 0$ by (2.3)-(2.5) and so $p_d^H, p_d^L > 0$ from (2.8)-(2.9); but then, by (2.6)-(2.7), $p_b^H = p_b^L = 0$; this implies that the marginal utility of wealth for a doctor is infinite and so an agent would prefer an insurance contract that gives him positive wealth if he is doctor, i.e., $\theta > -e$.

So we know that a doctor will have positive wealth. In combination with $x_d^H \ge x_d^L$, this implies $x_d^H > 0$. Note that the rest of the proof of Lemma 1 applies as long as $x_d^H \ge 1$. Suppose $x_d^H < 1$. It follows from (2.3) and (2.5) that $x_d^L > 0$. In other word doctors and builders both buy the high state Arrow security. But then from (*) and (**) the marginal utility of wealth of doctors equals π / q_H while the marginal utility of wealth of builders is $\pi / q_H p_b^H$. There are two cases: $p_b^H < 1$ and $p_b^H = 1$. If $p_b^H < 1$ the marginal utility of wealth of doctors is higher, implying that each doctor will want the maximum redistribution of wealth. Thus in this case a necessary condition for equilibrium is $\theta = e$. But then a doctor's budget constraint in combination with $x_d^H \ge x_d^L$ implies $x_d^H \ge 1$, whereas we have assumed $x_d^H < 1$. This is a contradiction. Hence $p_b^H = 1$. But then we must have $x_d^H \ge 1$ from (2.6), which is again a contradiction.

We have established that $x_d^H \ge 1$, which implies $p_b^H = 1$ by (2.6).

We may conclude that all of Lemma 1 holds: $p_d^H = p_b^H = 1$, $x_d^H \ge 1$.

Now turn to the low state. Consider the proof of Proposition 2. Nothing in the logic of the proof depends on the relative amounts of the doctor's or builder's endowments, i.e., θ . In particular, the values of

$p_b^L, p_d^L, x_b^L, x_d^L, q_H, q_L$ are pinned down by the conditions that both technologies are used and that doctors hold both securities. Furthermore, builders prefer the high state security to the low state one. It follows that the marginal utility of wealth of doctors, π / q_H, equals that of builders, $\pi / q_H p_b^H$ (since $p_b^H = 1$). Thus, it is an equilibrium for no agent to purchase insurance: $\theta = 0$. (There may be other equilibria but all are equivalent to one in which there is no insurance If $\theta \neq 0$ doctors use their additional wealth to buy more high state Arrow securities.)