Ross Prize Presentation

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Liquidity Asset Pricing Model (LAPM)

States and prices:

- ullet ω the state of nature revealed at date 1
- $f(\omega)$ the probability of ω
- $s(\omega)f(\omega)$ the date-0 price of liquidity delivered in state ω (at date 1)

Firms:

- J firms indexed by j
- ullet Investments I_j at date 0 and $i_j(\omega) \leq I_j$ at date 1 in state ω
- Production technologies described by $\{\rho_{j0}(\omega), \rho_{j1}(\omega), \rho_{j}(\omega)\}$, where $\rho_{j0}(\omega)$ is pledgeable date-2 return, $\rho_{j1}(\omega)$ is total date-2 return, and $\rho_{j}(\omega)$ is reinvestment cost at date 1 all per unit of continuation investment $i_{j}(\omega)$. Key assumption: $\rho_{j1}(\omega) > \rho_{j0}(\omega)$ (non-pledgeable wedge)

Outside liquidity:

- Exogenously given assets $\{L_k\}$, k=1,...,K, providing liquidity $L_k(\omega) \geq 0$ in state ω .
- Aggregate supply of liquidity in state ω is

$$L_S(\omega) = \sum_{j} \rho_{j0}(\omega) i_j(\omega) + \sum_{k} L_k(\omega).$$
 (1)

ullet Aggregate demand for liquidity in state ω is

$$L_D(\omega) = \sum_{j} \rho_j(\omega) i_j(\omega). \tag{2}$$

Equilibrium

• Set of prices $s(\omega) \ge 1$, and firm plans $\{I_j, I_j(\omega)\}$ such that net aggregate liquidity demand by the corporate sector satisfies:

$$\sum_{j} [\rho_{j}(\omega) - \rho_{j0}(\omega)] i_{j}(\omega) \le \sum_{k} L_{k}(\omega), \quad \forall \ \omega.$$
 (3)

with an equality whenever $s(\omega)>1$.

• Given prices $\{s(\omega)\}$, firm j solves the following problem:

$$\max_{\{I_j,i_j(\cdot)\}} \quad \sum_{\omega} [\rho_{j1}(\omega) - \rho_j(\omega)] i_j(\omega) f(\omega) \tag{4}$$

subject to

(i)
$$\sum_{\omega} [\rho_{j0}(\omega) - \rho_j(\omega)] i_j(\omega) s(\omega) f(\omega) \ge I_j - A_j$$
 (5)

$$(ii)$$
 $0 \le i_j(\omega) \le I_j$, $\forall \omega$.



Main Theorem: Equilibrium exists and is constrained efficient.

Key idea: Instead of viewing firm as choosing a continuation scale $i_i(\omega)$ have it choose the demand for liquidity $\ell_i(\omega)$:

$$\ell_j(\omega) \equiv [\rho_j(\omega) - \rho_{j0}(\omega)]i_j(\omega). \tag{6}$$

Transforms model to a standard exchange economy without production.

Asset prices

Date-0 prices of exogenous assets.

$$q_{k} = \frac{\sum_{\omega} f(\omega) L_{k}(\omega) s(\omega)}{\sum_{\omega} f(\omega) L_{k}(\omega)} \ge 1$$
 (7)

• Date-0 prices of equity shares (defined as the amount that firm j can raise for investment at date 0 per unit of expected return at date 1):

$$q_{j} = \frac{\sum_{\omega} f(\omega) [\rho_{j0}(\omega) - \rho_{j}(\omega)] i_{j}(\omega) s(\omega)}{\sum_{\omega} f(\omega) [\rho_{j0}(\omega) - \rho_{j}(\omega)] i_{j}(\omega)}.$$
 (8)

Comments:

- Minor deviation from AD-type framework. Disciplined. Still some interesting implications with robust logic.
- Easy to expand on: more than one technology per firm, income shocks, etc (Jean's book)
- Highlights simultaenous liquidity and risk management
- Pricing of liquidity guide for optimal use of liquidity: (i) private sector, (ii) government, (iii) international liquidity
- State-contingent use of collateral proxied by repo market
- No link between consumers and producers, no dynamics (but see LAPM paper)