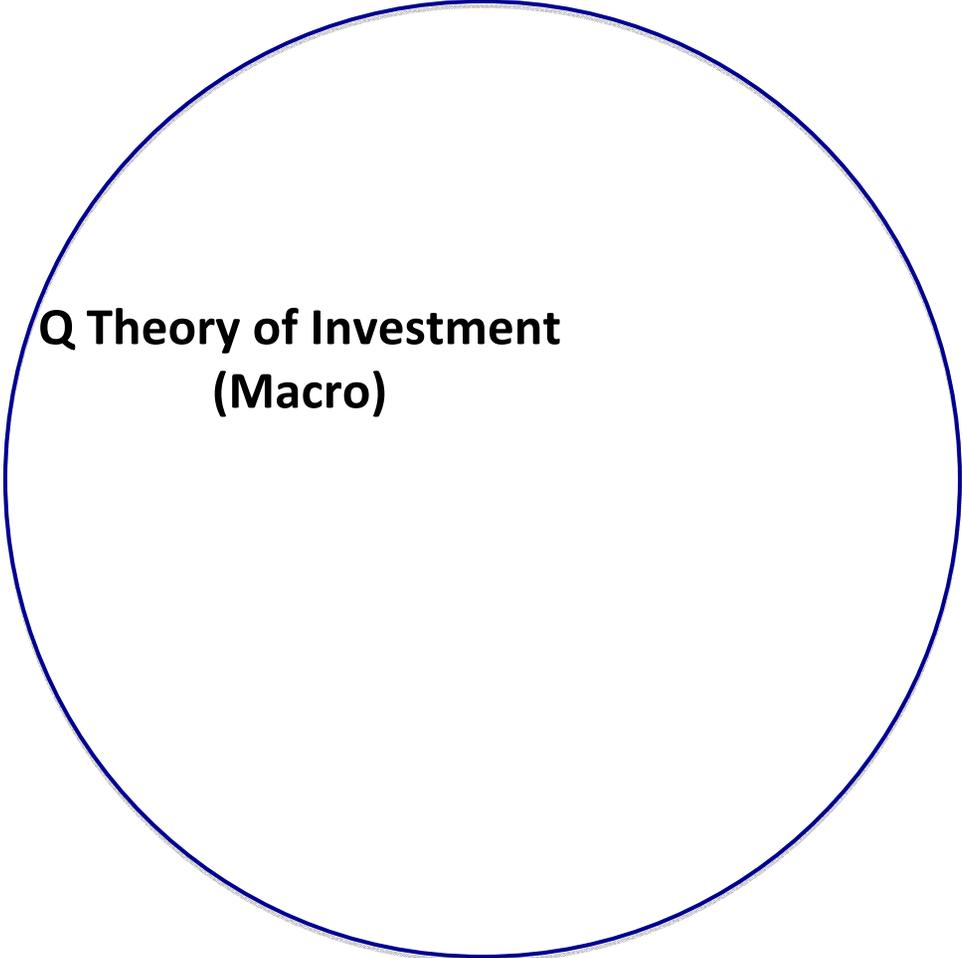# A UNIFIED THEORY OF TOBIN'S q, CORPORATE INVESTMENT, FINANCING, AND RISK MANAGEMENT
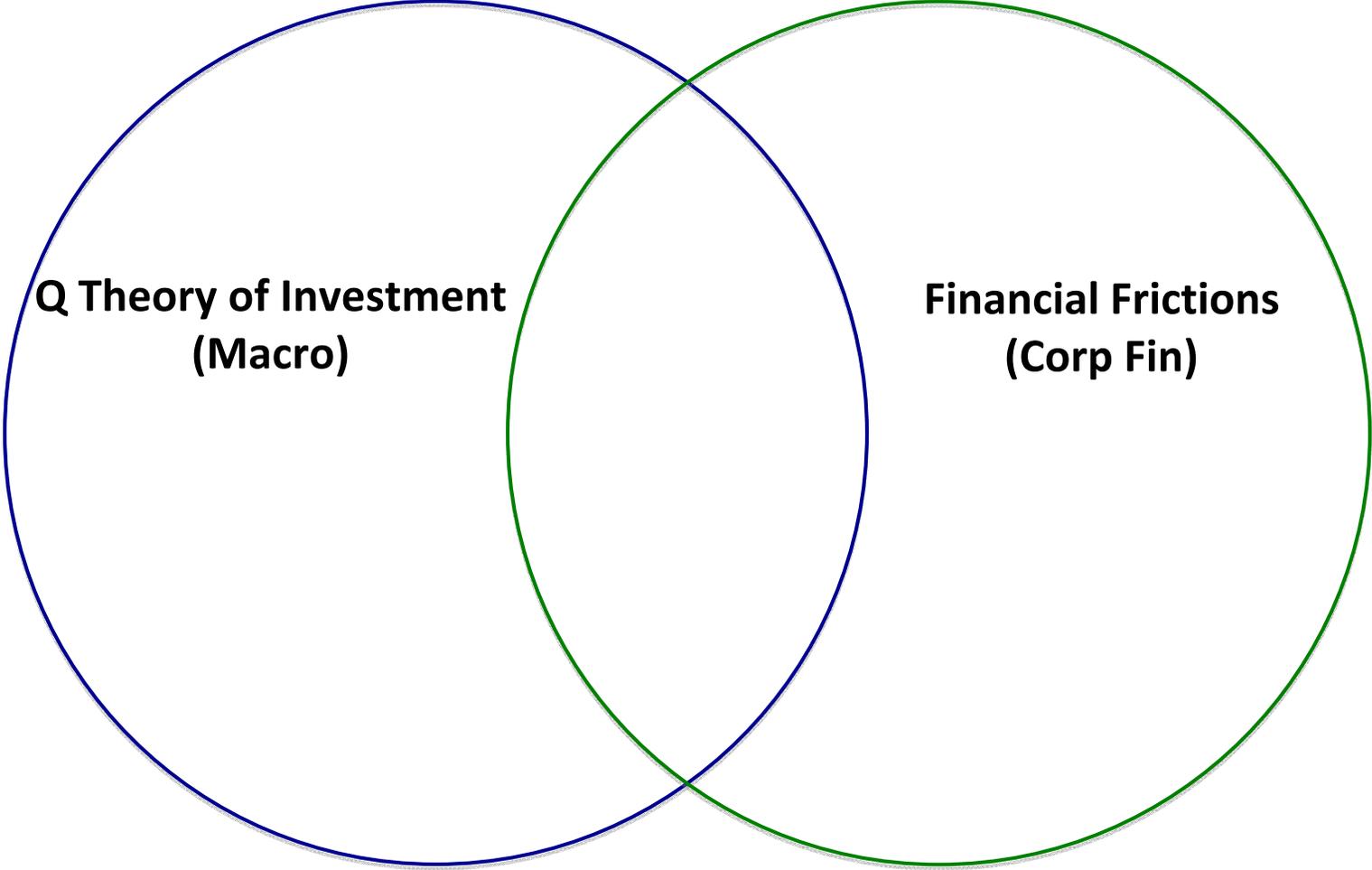
Patrick Bolton, Columbia, CEPR, and NBER

Hui Chen, MIT

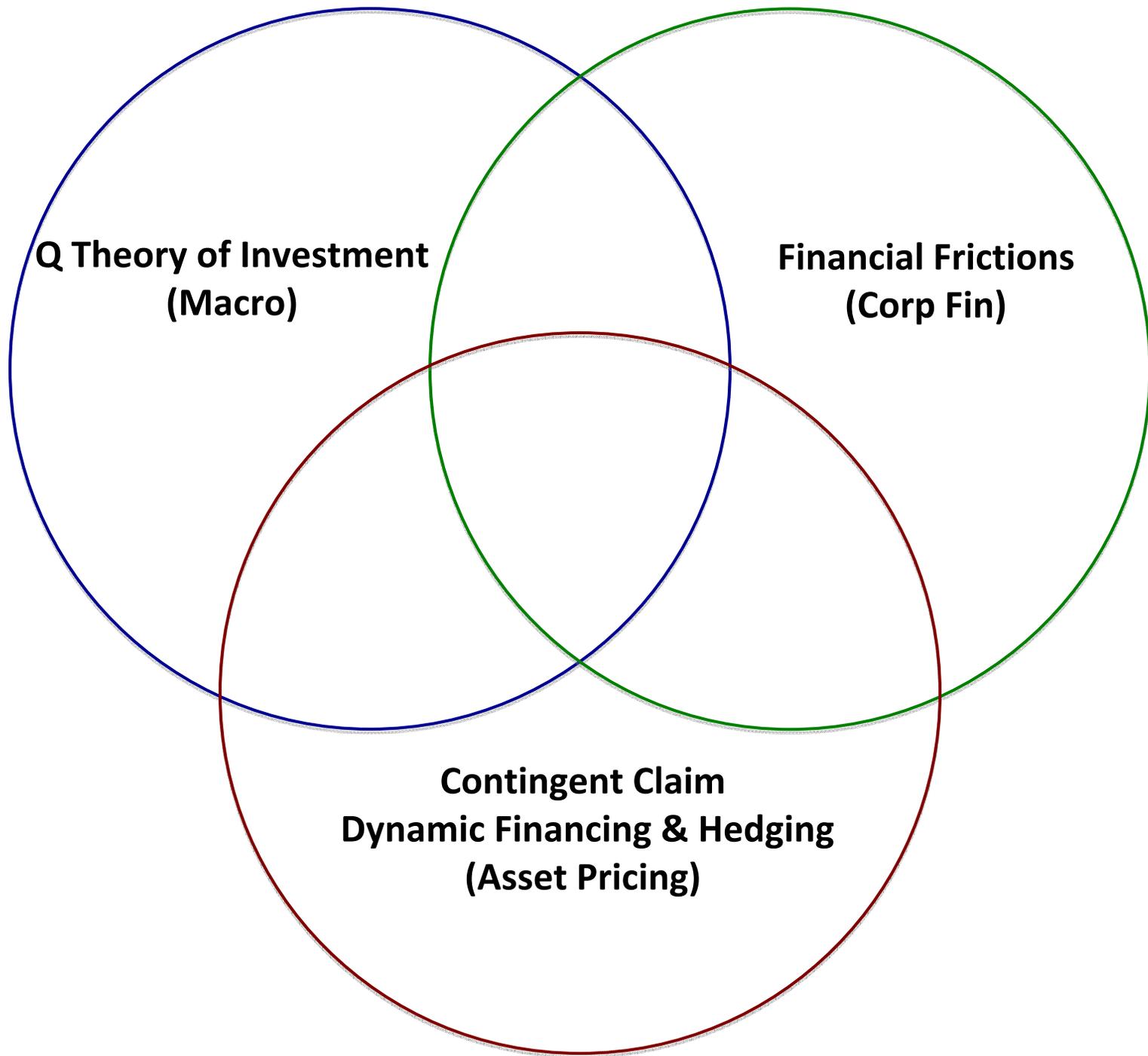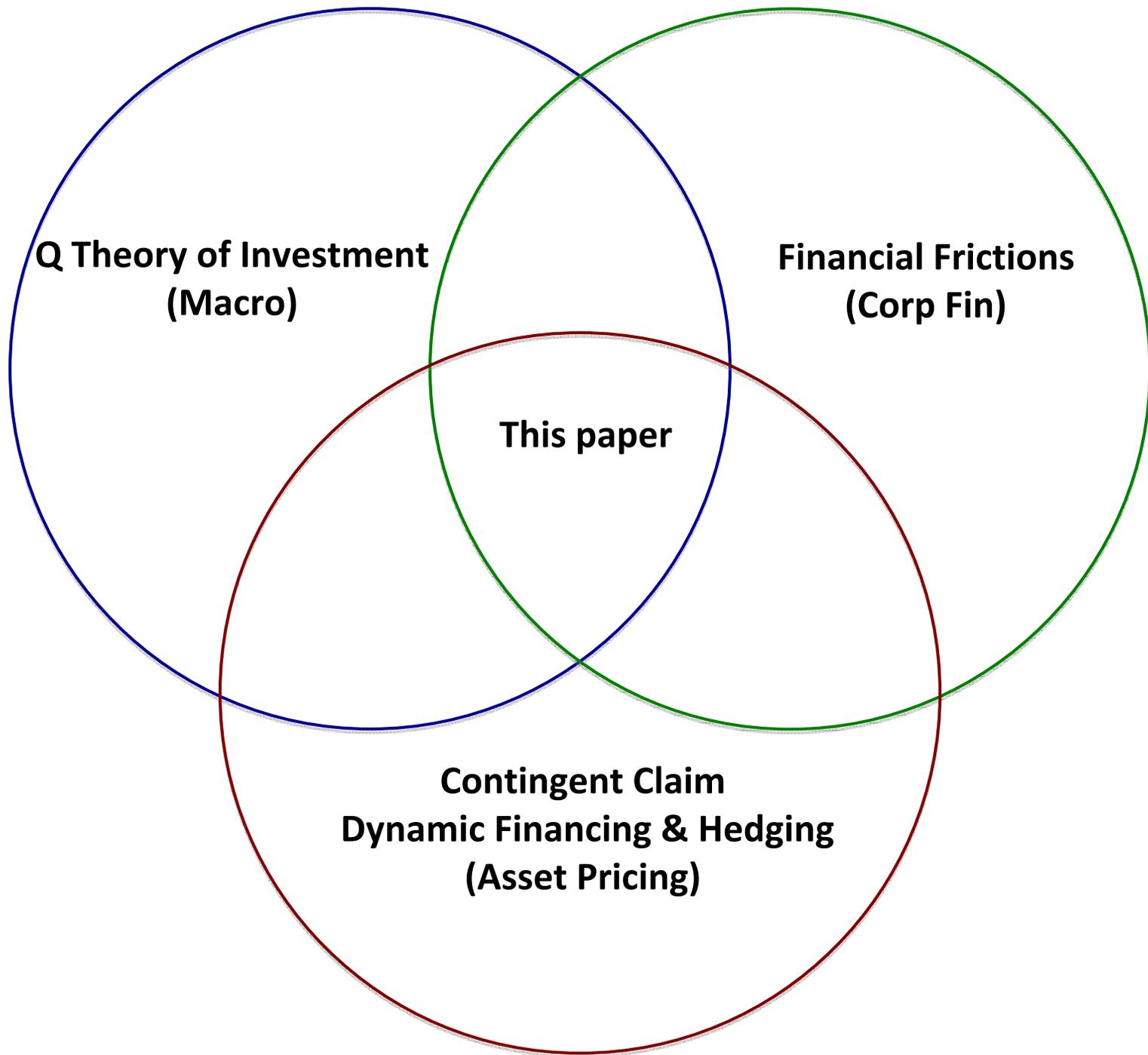Neng Wang, Columbia and NBER

Q Theory of Investment (Macro)

Q Theory of Investment
(Macro)

Financial Frictions
(Corp Fin)

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Contingent Claim
Dynamic Financing & Hedging
(Asset Pricing)

Q Theory of Investment
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Financial Frictions
(Corp Fin)

This paper

Contingent Claim
Dynamic Financing & Hedging
(Asset Pricing)

MODEL DESIGN

- constant investment opportunity (with a benchmark q theory)

- constant financing opportunity

- the interaction

Model Sketch: Physical Production

- capital accumulation:

$$dK_t = (I_t - \delta K_t)dt$$

- output:

$$dY_t = K_t \left(\mu dt + \sigma dZ_t \right)$$

- cost of investing:

$$\left(I_t + G(I_t, K_t) \right) dt.$$

- adjustment cost $G(I, K) = g(I/K)K$, where

$$g(i) = \frac{\theta}{2} i^2.$$

MODEL SKETCH

- Dynamics for cash W: $\quad dW_t = \text{change} \quad \text{in} \quad \text{cash} =$
  - CF from operations (production)

Model Sketch

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- Precautionary motive for holding cash
- Cost of holding cash: λ
- Can pay out cash to investors at any time: 1 for 1

Model Sketch

- Option to liquidate
  - collect lK from capital and W from cash account

MODEL SKETCH

- ▶ Option to liquidate
  - ▶ collect lK from capital and W from cash account

- ▶ Option to issue external equity
  - ▶ fixed cost
  - ▶ variable cost

Model Sketch

- Option to liquidate
  - collect lK from capital and W from cash account

- Option to issue external equity
  - fixed cost
  - variable cost

- Credit line
  - credit limit: cK
  - interest rate: $r + \alpha$

FOUR (OPTIMALLY DIVIDED) REGIONS

- payout region
- cash region ($w > 0$)
- credit region ($w < 0$)
- equity financing (liquidation) region

SOLUTION SKETCH

- Firm value maximization: $P(K, W)$

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- Homogeneity: $P(K, W) = p(w) \times K$ where $w = W/K$

- Economics: stochastic balanced capital accumulation with financial frictions

SOLUTION SKETCH

▶ Investment first-order condition:

$$\text{marginal cost of production} = \frac{P_K(K, W)}{P_W(K, W)}$$

▶ marginal value of liquidity (marginal cost of funds):
$P_W(K, W) = p'(w)$

VALUE AND INVESTMENT

- firm value-capital ratio $p(w)$:

$$
\begin{aligned}
rp(w) =\ & (i(w) - \delta)\,(p(w) - wp'(w)) \\
& + ((r - \lambda)w + \mu - i(w) - g(i(w)))\,p'(w) \\
& + \frac{\sigma^2}{2}p''(w)\,.
\end{aligned}
$$

- investment-capital ratio:

$$
i(w) = \frac{1}{\theta}\left(\frac{p(w)}{p'(w)} - w - 1\right).
$$

- Optimal composition of the balance sheet: W and K

AVERAGE AND MARGINAL q

- Optimal composition of the balance sheet: W and K
- Average q:

$$q_a(w) = \frac{P(K,W) - W}{K} = p(w) - w$$

AVERAGE AND MARGINAL q

- Optimal composition of the balance sheet: W and K
- Average q:

$$q_a(w) = \frac{P(K, W) - W}{K} = p(w) - w$$

- Marginal q:

$$q_m(w) = \frac{dP(K, W)}{dK} = q_a(w) - (p'(w) - 1)\, w$$

PARAMETER VALUES

risk-free rate	$r = 6\%$
mean productivity shock (risk-neutral)	$\mu = 18\%$
volatility of productivity shock	$\sigma = 9\%$
rate of depreciation	$\delta = 10.07\%$
adjustment cost	$\theta = 1.5$
liquidation value	$l = 0.9$
cash-carrying cost	$\lambda = 1\%$
proportional financing cost	$\gamma = 6\%$
fixed financing cost	$\phi = 1\%$

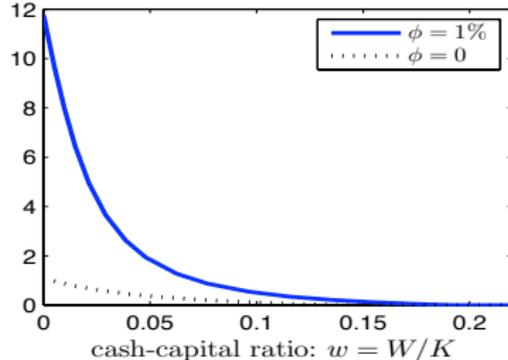- Benchmark: $i^{FB} = 15.1\%$ and $q^{FB} = 1.23$.

LIQUIDATION

A. firm value-capital ratio: $p(w)$
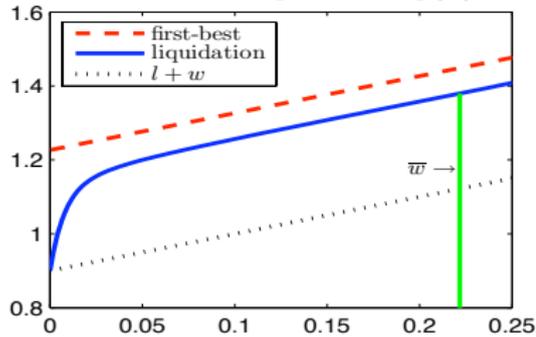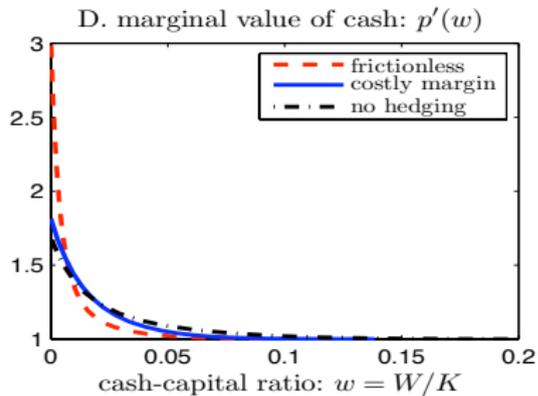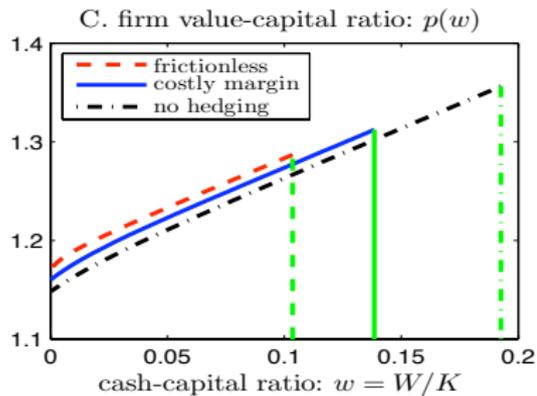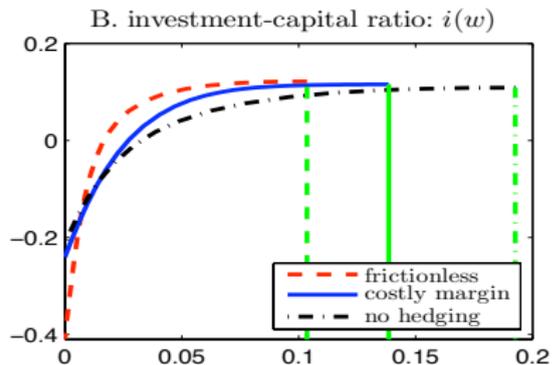
B. marginal value of cash: $p'(w)$

C. investment-capital ratio: $i(w)$
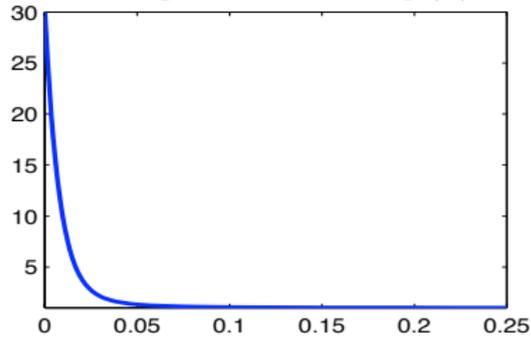
D. investment-cash sensitivity: $i'(w)$
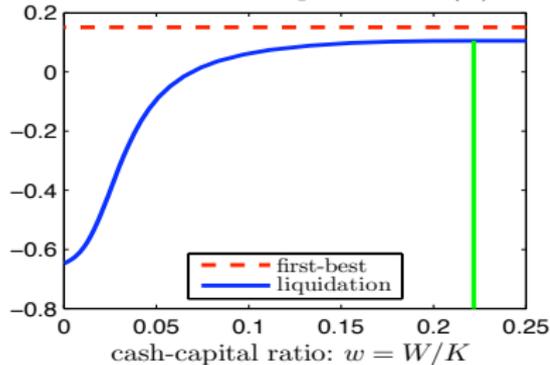
cash-capital ratio: $w = W/K$
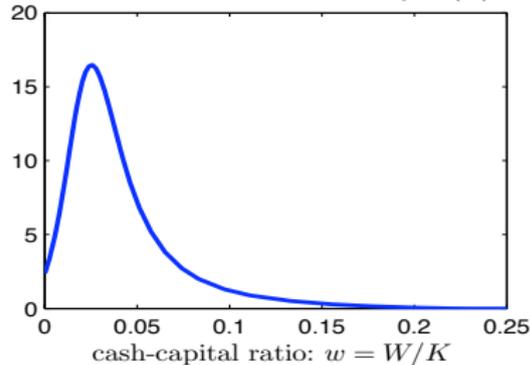
OPTIMAL FINANCING

A. firm value-capital ratio: $p(w)$
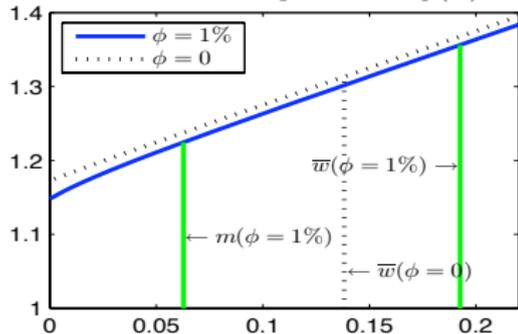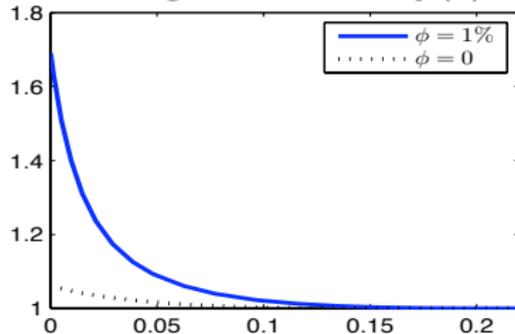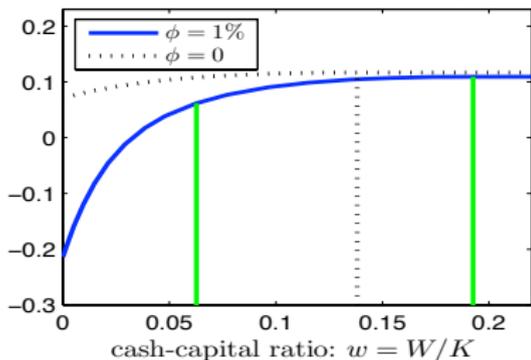
B. marginal value of cash: $p'(w)$

C. investment-capital ratio: $i(w)$

D. investment-cash sensitivity: $i'(w)$

cash-capital ratio: $w = W/K$

DYNAMIC HEDGING

A. hedge ratio: $\psi(w)$

B. investment-capital ratio: $i(w)$

C. firm value-capital ratio: $p(w)$

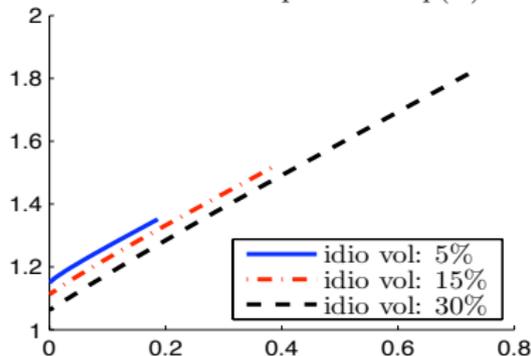D. marginal value of cash: $p'(w)$
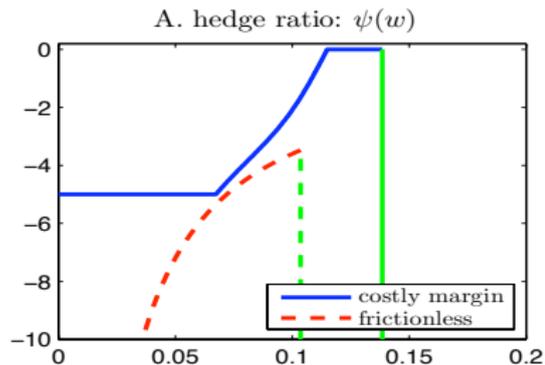
cash-capital ratio: $w = W/K$
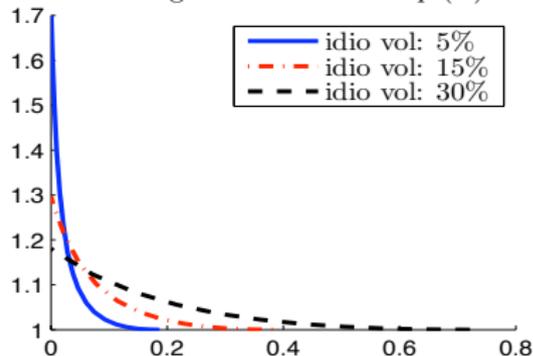
- Conditional β:

$$\beta(w) = \frac{\rho\sigma}{\sigma_m} \frac{p'(w)}{q_a(w) + w},$$

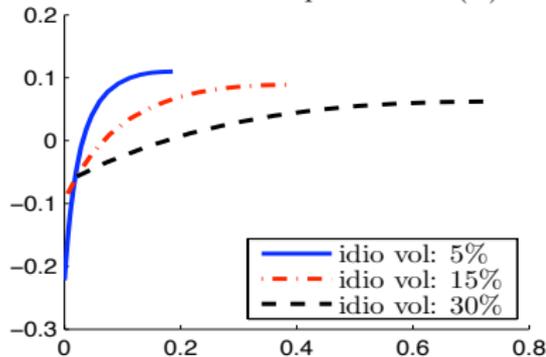# IDIOSYNCRATIC VOLATILITY AND BETA

A. firm value-capital ratio: $p(w)$
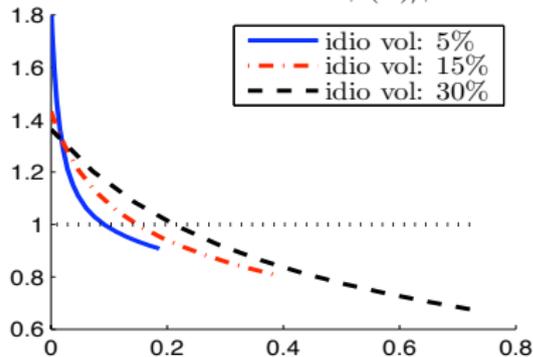
B. marginal value of cash: $p'(w)$
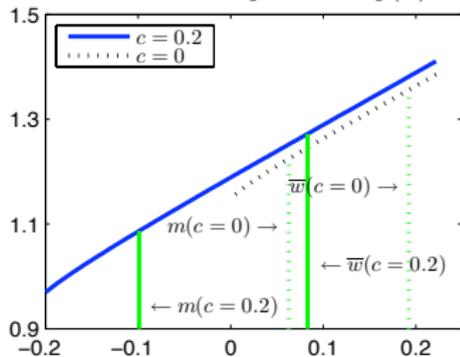
C. investment-capital ratio: $i(w)$

D. conditional beta: $\beta(w)/\beta^{FB}$

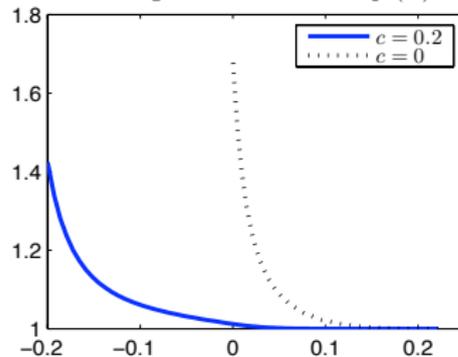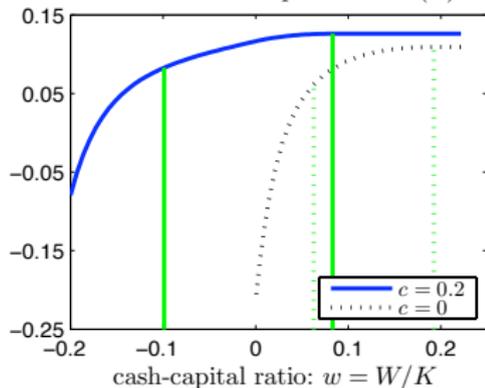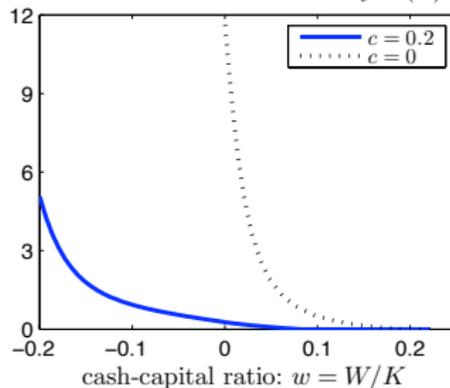idio vol: 5%
idio vol: 15%
idio vol: 30%

CREDIT LINE

A. firm value-capital ratio: $p(w)$

B. marginal value of cash: $p'(w)$

C. investment-capital ratio: $i(w)$

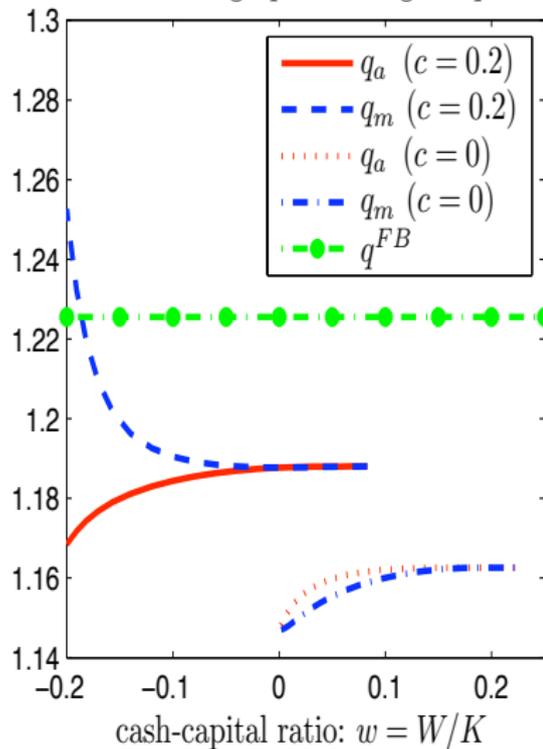D. investment-cash sensitivity: $i'(w)$

cash-capital ratio: $w = W/K$

$\overline{w}(c=0) \rightarrow$

$m(c=0) \rightarrow$

$\leftarrow \overline{w}(c=0.2)$

$\leftarrow m(c=0.2)$

$c = 0.2$
$c = 0$

INVESTMENT AND q

A. average q and marginal q
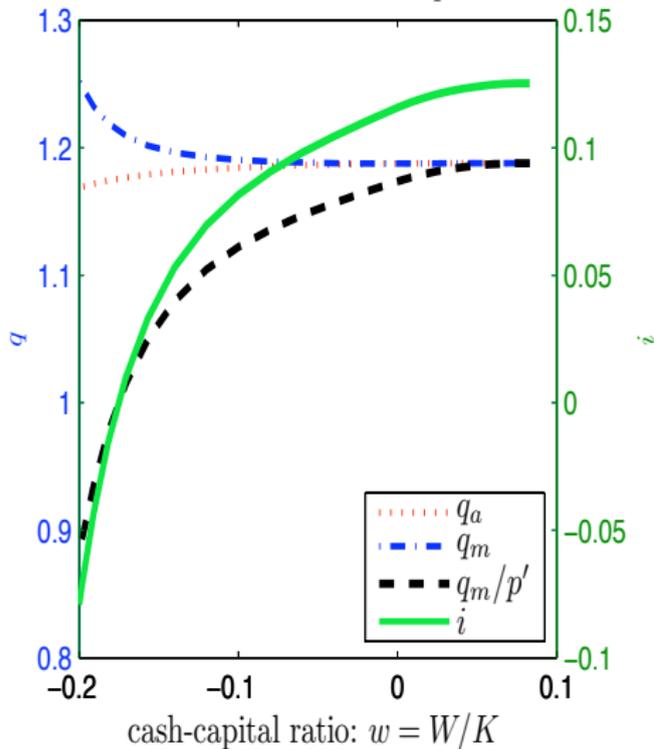
- q_a $(c=0.2)$
- q_m $(c=0.2)$
- q_a $(c=0)$
- q_m $(c=0)$
- q^{FB}

cash-capital ratio: $w = W/K$

B. investment and q

- q_a
- q_m
- q_m/p'
- i

cash-capital ratio: $w = W/K$

Main Results

▶ Investment first-order equation

$$\text{marginal cost of investing} = \frac{\text{marginal } q}{\text{marginal cost of financing}}$$

MAIN RESULTS

- Investment first-order equation

$$\text{marginal cost of investing} = \frac{\text{marginal } q}{\text{marginal cost of financing}}$$

- financing: no target cash level, target leverage;
  financing regions:
    - payout
    - cash
    - credit
    - equity financing (or liquidation)

Main Results (continued)

- risk management: cash, hedging, and asset sales
- firm value and risk/return: idiosyncratic and systematic risks
- methodology: tractable dynamic economic framework